On the Usefulness of the Evolution Strategies' Self-Adaptation Mechanism to Handle Constraints in Global Optimization*

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Abstract

In this paper, we argue that the original self-adaptation mechanism of the Evolution Strategies is useful by itself to handle constraints in global optimization. We show how using just three simple comparison criteria the simple Evolution Strategy can be led to the feasible region of the search space and find the global optimum solution (or a very good approximation of it). Different Evolution Strategies including $(\mu+1)-ES$ and $(\mu \overset{+}{,} \lambda)-ES$ with or without correlated mutation were implemented. Such approaches have been tested using the well-known test suit of Michalewicz and Schnoenauer and four engineering problems. The results are discussed and some conclusions are drawn.

1 Introduction

Evolution Strategies (ES) have been widely used to solve global optimization problems [32, 17, 16, 11, 24, 15, 9, 7, 5, 1, 2, 3]. Moreover, there is a theoretical background that supports ES convergence [30, 6, 12, 8]. However, as other Evolutionary Algorithms (Evolutionary Programming and Genetic Algorithms), ES lack an explicit mechanism to deal with constrained search spaces. The recombination and mutation operators

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cannot distinguish between feasible and infeasible solutions. Therefore, several approaches have been suggested in the literature to allow Evolutionary Algorithms (EAs) to deal with constrained problems [13].

The most common approach adopted to deal with constrained search spaces is the use of penalty functions. When using a penalty function, the amount of constraint violation is used to punish or "penalize" an infeasible solution so that feasible solutions are favored by the selection process. Despite the popularity of penalty functions, they have several drawbacks from which the main one is that they require a careful fine tuning of the penalty factors that accurately estimates the degree of penalization to be applied as to approach efficiently the feasible region [33, 13].

There are also studies about using multiobjective concepts to handle constraints in EAs [22]. These approaches find or approximate the optimal solution with less fitness function evaluations than other competitive approaches like the Homomorphous Maps of Koziel and Michalewicz [21].

Two of the most recent techniques to handle constraints in EAs found in the literature, the Stochastic Ranking by Runarsson & Yao [28] and the Adaptive Segregational Contraint Handling Evolutionary Algorithm (ASCHEA) by Hamida & Schoenauer [18, 19] are both based on an ES. The quality and consistency of the reported results of both approaches are very good. This suggests that ES's original self-adaptation mechanism might help the EA to deal with constrained search spaces. Thus, we decided to compare three different types of ES ($(\mu + 1)$, $(\mu + \lambda)$ and (μ, λ)) using just three simple comparison criteria to solve the well-known benchmark for global nonlinear optimization proposed by Michalewicz and Schoenauer [23] and extended by Runarsson & Yao [28]. We also analyze the uselfulness of the correlated mutation in population-based ES.

This paper is organized as follows: In Section 2 we briefly describe the main concepts of ES. In Section 3, we provide an explanation of the simple constraint handling approach adopted in this work. After that, in Section 4, we present the results obtained of our experiments. The discussion of such results is on Section 5. Finally, in Section 6 we provide our conclusions and some possible paths of future research.

2 Evolution Strategies

ES were proposed by Bienert, Rechenberg and Schwefel. They used them to solve hydrodynamical problems [26, 31]. The first ES version was the $(1 + 1)$-ES which uses just one individual that is mutated using a normal distributed random number with mean zero and an identical standard deviation for each decision variable. The best solution between the parent and the offspring is chosen and the other one is eliminated. Rechenberg derived a convergence rate theory and proposed a rule for changing the standard deviation of mutations called the "$1/5$-success rule" [27].

The first multimembered ES was the $(\mu + 1)$-ES, which was designed by Rechenberg and is described in detail in [8]. In this approach, μ parent solutions recombine to generate one offspring. This solution is also mutated and, if it is better, it will replace the worst parent solution. Note however that the $(\mu+1)$-ES has not been too popular in the literature. However, it provided the transition to the state-of-the-art multimembered

ES.

The $(\mu+\lambda)$-ES and the (μ,λ)-ES were proposed by Schwefel [29]. In the first one, the best μ individuals out of the union of the μ original parents and their λ offspring will survive for the next generation. On the other hand, in the (μ,λ)-ES the best μ will only be selected from the λ offspring.

The $(\mu+\lambda)$-ES uses an implicit elitist mechanism and solutions can survive more than one generation. Meanwhile, in the (μ,λ)-ES solutions only survive one generation. Instead of the "$1/5$-success rule", each individual includes a standard deviation value for each decision variable. Moreover, for each combination of two standard deviation values, a rotation angle is included. These angles are used to perform a correlated mutation. This mutation allows each individual to look for a search direction. The standard deviations and the angles of each individual are called strategy parameters. They are also recombined and mutated. A $(\mu+\lambda)$-ES or (μ,λ)-ES individual can be seen as follows: $a(i)(\vec{x},\vec{\sigma},\vec{\theta})$, where i is the number of individual in the population, $\vec{x}\in\Re^n$ is a vector of n decision variables, $\vec{\sigma}$ is a vector of n standard deviations and $\vec{\theta}$ is a vector of $n(n-1)/2$ rotation angles where $\theta_i\in[-\pi,\pi]$.

There are two types of recombination: sexual (two individuals) and panmictic (more than two solutions). There is a variety of recombinations forms for both types: discrete, intermediate and generalized [6].

The mutation operator works on the decision variables and also on the strategy parameters. The mutation is calculated in the following way:

$$\sigma_i' = \sigma_i \cdot exp(\tau' \cdot N(0,1) + \tau \cdot N_i(0,1)) \tag{1}$$

$$\theta_j' = \theta_j + \beta \cdot N_j(0,1) \tag{2}$$

$$\vec{x}' = \vec{x} + \vec{N}(\vec{0}, C(\vec{\sigma'}, \vec{\theta'})) \tag{3}$$

where τ and τ' are interpreted as "learning rates" and are defined by Schwefel [6] as: $\tau = (\sqrt{2\sqrt{n}})^{-1}$ and $\tau' = (\sqrt{2n})^{-1}$ and $\beta \approx 0.0873$.

Some authors use correlated mutation, whereas others prefer to use a non-correlated mutation. In this way, the computational effort and the memory space used by each individual gets lower.

If a non-correlated mutation is used, the mutation expressions are:

$$\sigma_i' = \sigma_i \cdot exp(\tau' \cdot N(0,1) + \tau \cdot N_i(0,1)) \tag{4}$$

$$x_i' = x_i + \sigma_i' \cdot N_i(0,1) \tag{5}$$

The general ES algorithm is detailed in figure 1.

3 Constraint-Handling Approach

As it was discussed in Section 1, we argue that the natural self-adaptation mechanism of the ES is useful to bias a evolutionary search through a constrained space. In this way, just three comparison criteria are used to select the best individuals from one generation:

```
Begin
    t=0
    Create μ random solutions for the initial population.
    Evaluate all u individuals
    Assign a fitness value to all μ individuals
    For t=1 to MAX_GENERATIONS Do
        Produce λ offspring by recombination of the μ parents
        Mutate each child
        Evaluate all λ offspring
        Assign a fitness value to all λ individuals
        If Selection = "+" Then
            Select the best μ individuals from the μ + λ individuals
        Else
            Select the best μ individuals from the λ individuals
        End If
    End For
End
```

Figure 1: ES general algorithm

- Between 2 feasible solutions, the one with the higher fitness value is preferred.
- If one solution is feasible and the other one is infeasible, the feasible one is preferred.
- If both solutions are infeasible, the one with the lowest sum of constraint violation is preferred.

4 Experiments and Results

To evaluate the performance of the techniques selected, we decided to use the well-known benchmark proposed in [23] plus four engineering design problems used in [14]. The full description of the seventeen test functions is the following:

1. **Problem 1: (g01)**:

   Minimize:
   $$f(\vec{x}) = 5\sum_{i=1}^{4} x_i - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i \tag{6}$$

   subject to:

$$
\begin{aligned}
g_1(\vec{x}) &= 2x_1 + 2x_2 + x_{10} + x_{11} - 10 \leq 0 \\
g_2(\vec{x}) &= 2x_1 + 2x_3 + x_{10} + x_{12} - 10 \leq 0 \\
g_3(\vec{x}) &= 2x_2 + 2x_3 + x_{10} + x_{12} - 10 \leq 0 \\
g_4(\vec{x}) &= -8x_1 + x_{10} \leq 0 \\
g_5(\vec{x}) &= -8x_2 + x_{11} \leq 0 \\
g_6(\vec{x}) &= -8x_3 + x_{12} \leq 0 \\
g_7(\vec{x}) &= -2x_4 - x_5 + x_{10} \leq 0 \\
g_8(\vec{x}) &= -2x_6 - x_7 + x_{11} \leq 0 \\
g_9(\vec{x}) &= -2x_8 - x_9 + x_{12} \leq 0
\end{aligned} \tag{7}
$$

where $0 \leq x_i \leq 1$ $(i = 1, \ldots, 9)$ $0 \leq x_i \leq 100$ $(i = 10, 11, 12)$ and $0 \leq x_{13} \leq 1$. The global optimum is at $x^* = (1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 3, 3, 1)$ where $f(x^*) = -15$. Constraints g_1, g_2, g_3, g_4, g_5 and g_6 are active.

2. **Problem 2: (g02)**:

   Maximize:

$$
f(\vec{x}) = \left| \frac{\sum_{i=1}^{n} \cos^4(x_i) - 2 \prod_{i=1}^{n} \cos^2(x_i)}{\sqrt{\sum_{i=1}^{n} i x_i^2}} \right| \tag{8}
$$

   subject to:

$$
\begin{aligned}
g_1(\vec{x}) &= 0.75 - \prod_{i=1}^{n} x_i \leq 0 \\
g_2(\vec{x}) &= \sum_{i=1}^{n} x_i - 7.5n \leq 0
\end{aligned} \tag{9}
$$

   where $n = 20$ and $0 \leq x_i \leq 10$ $(i = 1, \ldots, n)$. The global maximum is unknown; the best reported solution is [28] $f(x^*) = 0.803619$. Constraint g_1 is close to being active ($g_1 = -10^{-8}$).

3. **Problem 3: (g03)**:

   Maximize:

$$
f(\vec{x}) = \left(\sqrt{n}\right)^n \prod_{i=1}^{n} x_i \tag{10}
$$

   subject to:

$$
h(\vec{x}) = \sum_{i=1}^{n} x_i^2 - 1 = 0 \tag{11}
$$

   where $n = 10$ and $0 \leq x_i \leq 1$ $(i = 1, \ldots, n)$. The global maximum is at $x_i^* = 1/\sqrt{n}$ $(i = 1, \ldots, n)$ where $f(x^*) = 1$.

4. **Problem 4: (g04)**:

   Minimize:

$$f(\vec{x}) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \quad (12)$$

   subject to:

$$\begin{aligned}
g_1(\vec{x}) &= 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5 - 92 \leq 0 \\
g_2(\vec{x}) &= -85.334407 - 0.0056858x_2x_5 - 0.0006262x_1x_4 + 0.0022053x_3x_5 \leq 0 \\
g_3(\vec{x}) &= 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 - 110 \leq 0 \\
g_4(\vec{x}) &= -80.51249 - 0.0071317x_2x_5 - 0.0029955x_1x_2 - 0.0021813x_3^2 + 90 \leq 0 \\
g_5(\vec{x}) &= 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 - 25 \leq 0 \\
g_6(\vec{x}) &= -9.300961 - 0.0047026x_3x_5 - 0.0012547x_1x_3 - 0.0019085x_3x_4 + 20 \leq 0
\end{aligned} \quad (13)$$

   where: $78 \leq x_1 \leq 102$, $33 \leq x_2 \leq 45$, $27 \leq x_i \leq 45$ $(i = 3, 4, 5)$. The optimum solution is $x^* = (78, 33, 29.995256025682, 45, 36.775812905788)$ where $f(x^*) = -30665.539$. Constraints g_1 y g_6 are active.

5. **Problem 5: (g5)**

   Minimize:

$$f(\vec{x}) = 3x_1 + 0.000001x_1^3 + 2x_2 + (0.000002/3)x_2^3 \quad (14)$$

   subject to:

$$\begin{aligned}
g_1(\vec{x}) &= -x_4 + x_3 - 0.55 \leq 0 \\
g_2(\vec{x}) &= -x_3 + x_4 - 0.55 \leq 0 \\
h_3(\vec{x}) &= 1000\sin(-x_3 - 0.25) + 1000\sin(-x_4 - 0.25) + 894.8 - x_1 = 0 \\
h_4(\vec{x}) &= 1000\sin(-x_3 - 0.25) + 1000\sin(x_3 - x_4 - 0.25) + 894.8 - x_2 = 0 \\
h_5(\vec{x}) &= 1000\sin(-x_4 - 0.25) + 1000\sin(x_4 - x_3 - 0.25) + 1294.8 = 0
\end{aligned} \quad (15)$$

   where $0 \leq x_1 \leq 1200$, $0 \leq x_2 \leq 1200$, $-0.55 \leq x_3 \leq 0.55$, and $-0.55 \leq x_4 \leq 0.55$. The best known solution is $x^* = (679.9453, 1026.067, 0.1188764, -0.3962336)$ where $f(x^*) = 5126.4981$.

6. **Problem 6: (g6)**

   Minimize:

$$f(\vec{x}) = (x_1 - 10)^3 + (x_2 - 20)^3 \quad (16)$$

   subject to:

$$\begin{aligned} g_1(\vec{x}) &= -(x_1-5)^2-(x_2-5)^2+100 \leq 0 \\ g_2(\vec{x}) &= (x_1-6)^2+(x_2-5)^2-82.81 \leq 0 \end{aligned} \tag{17}$$

where $13 \leq x_1 \leq 100$ and $0 \leq x_2 \leq 100$. The optimum solution is $x^* = (14.095, 0.84296)$ where $f(x^*) = -6961.81388$. Both constraints are active.

7. **Problem 7: (g7)**

   Minimize:

$$\begin{aligned} f(\vec{x}) = \; & x_1^2+x_2^2+x_1x_2-14x_1-16x_2+(x_3-10)^2+4(x_4-5)^2+(x_5-3)^2 \\ & +2(x_6-1)^2+5x_7^2+7(x_8-11)^2+2(x_9-10)^2 \\ & +(x_{10}-7)^2+45 \end{aligned} \tag{18}$$

   Subject to:

$$\begin{aligned} g_1(\vec{x}) &= -105+4x_1+5x_2-3x_7+9x_8 \leq 0 \\ g_2(\vec{x}) &= 10x_1-8x_2-17x_7+2x_8 \leq 0 \\ g_3(\vec{x}) &= -8x_1+2x_2+5x_9-2x_{10}-12 \leq 0 \\ g_4(\vec{x}) &= 3(x_1-2)^2+4(x_2-3)^2+2x_3^2-7x_4-120 \leq 0 \\ g_5(\vec{x}) &= 5x_1^2+8x_2+(x_3-6)^2-2x_4-40 \leq 0 \\ g_6(\vec{x}) &= x_1^2+2(x_2-2)^2-2x_1x_2+14x_5-6x_6 \leq 0 \\ g_7(\vec{x}) &= 0.5(x_1-8)^2+2(x_2-4)^2+3x_5^2-x_6-30 \leq 0 \\ g_8(\vec{x}) &= -3x_1+6x_2+12(x_9-8)^2-7x_{10} \leq 0 \end{aligned} \tag{19}$$

   where $-10 \leq x_i \leq 10$ $(i = 1, \ldots, 10)$. The global optimum is $x^* = (2.171996, 2.363683, 8.773926, 5.095984, 0.$ $9.828726, 8.280092, 8.375927)$ where $f(x^*) = 24.3062091$. Constraints g_1, g_2, g_3, g_4, g_5 and g_6 are active.

8. **Problem 8: (g8)**

   Maximize:

$$f(\vec{x}) = \frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1+x_2)} \tag{20}$$

   subject to:

$$\begin{aligned} g_1(\vec{x}) &= x_1^2-x_2+1 \leq 0 \\ g_2(\vec{x}) &= 1-x_1+(x_2-4)^2 \leq 0 \end{aligned} \tag{21}$$

   where $0 \leq x_1 \leq 10$ and $0 \leq x_2 \leq 10$. The optimum solution is located at $x^* = (1.2279713, 4.2453733)$ where $f(x^*) = 0.095825$. The solutions is located within the feasible region.

9. **Problem 9: (g9)**

   Minimize:

$$
\begin{aligned}
f(\vec{x}) = \quad & (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 \\
& +10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7
\end{aligned}
\tag{22}
$$

   subject to:

$$
\begin{aligned}
g_1(\vec{x}) &= -127 + 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \leq 0 \\
g_2(\vec{x}) &= -282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \leq 0 \\
g_3(\vec{x}) &= -196 + 23x_1 + x_2^2 + 6x_6^2 - 8x_7 \leq 0 \\
g_4(\vec{x}) &= 4x_1^2 + x_2^2 - 3x_1x_2 + 2x_3^2 + 5x_6 - 11x_7 \leq 0
\end{aligned}
\tag{23}
$$

10. **Problem 10: (g10)** Minimize:

$$
f(\vec{x}) = x_1 + x_2 + x_3 \tag{24}
$$

   Subject to:

$$
\begin{aligned}
g_1(\vec{x}) &= -1 + 0.0025(x_4 + x_6) \leq 0 \\
g_2(\vec{x}) &= -1 + 0.0025(x_5 + x_7 - x_4) \leq 0 \\
g_3(\vec{x}) &= -1 + 0.01(x_8 - x_5) \leq 0 \\
g_4(\vec{x}) &= -x_1x_6 + 833.33252x_4 + 100x_1 - 83333.333 \leq 0 \\
g_5(\vec{x}) &= -x_2x_7 + 1250x_5 + x_2x_4 - 1250x_4 \leq 0 \\
g_6(\vec{x}) &= -x_3x_8 + 1250000 + x_3x_5 - 2500x_5 \leq 0
\end{aligned}
\tag{25}
$$

   where $100 \leq x_1 \leq 10000$, $1000 \leq x_i \leq 10000$, $(i = 2, 3)$, $10 \leq x_i \leq 1000$, $(i = 4, \ldots, 8)$. The global optimum is: $x^* = (579.3167, 1359.943, 5110.071, 182.0174, 295.5985, 217.9799, 286.4162, 395.5979)$, where $f(x^*) = 7049.3307$. g_1, g_2 and g_3 are active.

11. **Problem 11: (g11)**

   Minimize:

$$
f(\vec{x}) = x_1^2 + (x_2 - 1)^2 \tag{26}
$$

   subject to:

$$
h(\vec{x}) = x_2 - x_1^2 = 0 \tag{27}
$$

   where: $-1 \leq x_1 \leq 1$, $-1 \leq x_2 \leq 1$. The optimum solution is $x^* = (\pm 1/\sqrt{2}, 1/2)$ where $f(x^*) = 0.75$.

12. **Problem 12: (g12)**

Maximize:

$$f(\vec{x}) = \frac{100 - (x_1 - 5)^2 - (x_2 - 5)^2 - (x_3 - 5)^2}{100} \quad (28)$$

Subject to:

$$g_1(\vec{x}) = (x_1 - p)^2 + (x_2 - q)^2 + (x_3 - r)^2 - 0.0625 \leq 0 \quad (29)$$

where $0 \leq x_i \leq 10$ $(i = 1, 2, 3)$ and $p, q, r = 1, 2, \ldots, 9$. The feasible region of the search space consists of 9^3 disjointed spheres. A point (x_1, x_2, x_3) is feasible if and only if there exist p, q, r such the above inequality (29) holds. The global optimum is located at $x^* = (5, 5, 5)$ where $f(x^*) = 1$. The solution lies within the feasible region.

13. **Problem 13: (g13)**

Minimize:

$$f(\vec{x}) = e^{x_1 x_2 x_3 x_4 x_5} \quad (30)$$

subject to:

$$\begin{aligned} g_1(\vec{x}) &= x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0 \\ g_2(\vec{x}) &= x_2 x_3 - 5 x_4 x_5 = 0 \\ g_3(\vec{x}) &= x_1^3 + x_2^3 + 1 = 0 \end{aligned} \quad (31)$$

where $-2.3 \leq x_i \leq 2.3$ $(i = 1, 2)$ and $-3.2 \leq x_i \leq 3.2$ $(i = 3, 4, 5)$. The optimum solution is $x^* = (-1.717143, 1.595709, 1.827247, -0.7636413, -0.763645)$ where $f(x^*) = 0.0539498$.

14. **Problem 14: (Design of a Welded Beam)**

A welded beam is designed for minimum cost subject to constraints on shear stress (τ), bending stress in the beam (σ), buckling load on the bar (P_c), end deflection of the beam (δ), and side constraints [25]. There are four design variables as shown in Figure 2 [25]: h (x_1), l (x_2), t (x_3) and b (x_4).

The problem can be stated as follows:

Minimize:

$$f(\vec{x}) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2) \quad (32)$$

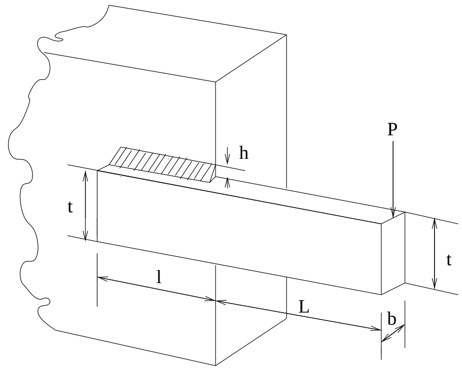

Figure 2: The welded beam used for problem 14 .

Subject to:

$$\begin{aligned}
g_1(\vec{x}) &= \tau(\vec{x}) - \tau_{max} \leq 0 \\
g_2(\vec{x}) &= \sigma(\vec{x}) - \sigma_{max} \leq 0 \\
g_3(\vec{x}) &= x_1 - x_4 \leq 0 \\
g_4(\vec{x}) &= 0.10471 x_1^2 + 0.04811 x_3 x_4 (14.0 + x_2) - 5.0 \leq 0 \\
g_5(\vec{x}) &= 0.125 - x_1 \leq 0 \\
g_6(\vec{x}) &= \delta(\vec{x}) - \delta_{max} \leq 0 \\
g_7(\vec{x}) &= P - P_c(\vec{x}) \leq 0
\end{aligned} \tag{33}$$

where

$$\tau(\vec{x}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}$$

$$\tau' = \frac{P}{\sqrt{2}x_1 x_2}, \tau'' = \frac{MR}{J}, M = P\left(L + \frac{x_2}{2}\right)$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$J = 2\left\{\sqrt{2}x_1 x_2 \left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$

$$\sigma(\vec{x}) = \frac{6PL}{x_4 x_3^2}, \delta(\mathbf{X}) = \frac{4PL^3}{E x_3^3 x_4}$$

$$P_c(\vec{x}) = \frac{4.013E\sqrt{\frac{x_3^2 x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right) \tag{34}$$

$$P = 6000\,lb, \;\; L = 14\,in, \;\; E = 30 \times 10^6\,psi, \;\; G = 12 \times 10^6\,psi$$

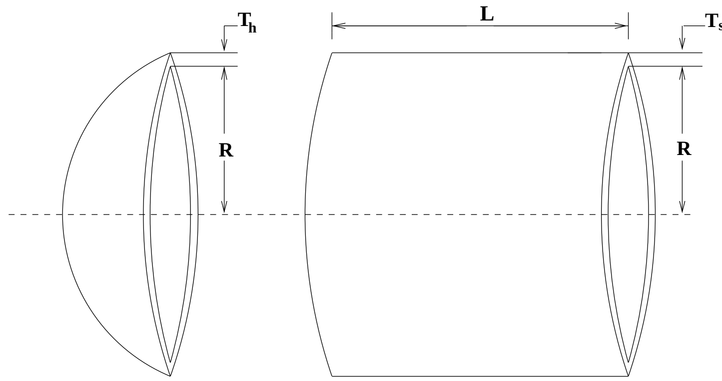

Figure 3: Center and end section of the pressure vessel used for problem 15.

$$\tau_{max} = 13,600\,psi, \;\; \sigma_{max} = 30,000\,psi, \;\; \delta_{max} = 0.25\,in$$

where $0.1 \leq x_1 \leq 2.0$, $0.1 \leq x_2 \leq 10.0$, $0.1 \leq x_3 \leq 10.0$ y $0.1 \leq x_4 \leq 2.0$.

15. **Problem 15: (Design of a Pressure Vessel)**

    A cylindrical vessel is capped at both ends by hemispherical heads as shown in Figure 3. The objective is to minimize the total cost, including the cost of the material, forming and welding. There are four design variables: T_s (thickness of the shell), T_h (thickness of the head), R (inner radius) and L (length of the cylindrical section of the vessel, not including the head). T_s and T_h are integer multiples of 0.0625 inch, which are the available thicknesses of rolled steel plates, and R and L are continuous. Using the same notation given by Kannan and Kramer [20], the problem can be stated as follows:

    Minimize :

$$f(\vec{x}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \tag{35}$$

    Subject to :

$$\begin{aligned} g_1(\vec{x}) &= -x_1 + 0.0193x_3 \leq 0 \\ g_2(\vec{x}) &= -x_2 + 0.00954x_3 \leq 0 \\ g_3(\vec{x}) &= -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1,296,000 \leq 0 \\ g_4(\vec{x}) &= x_4 - 240 \leq 0 \end{aligned} \tag{36}$$

    where $1 \leq x_1 \leq 99$, $1 \leq x_2 \leq 99$, $10 \leq x_3 \leq 200$ y $10 \leq x_4 \leq 200$.

16. **Problem 16: (Minimization of the Weight of a Tension/Compression String)**

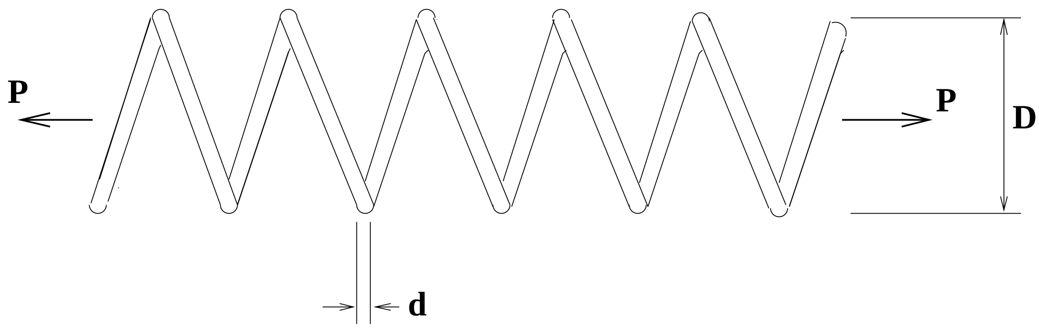

Figure 4: Tension/compression string used for problem 16.

This problem was described by Arora [4] and Belegundu [10], and it consists of minimizing the weight of a tension/compression spring (see Figure 4) subject to constraints on minimum deflection, shear stress, surge frequency, limits on outside diameter and on design variables. The design variables are the mean coil diameter D (x_2), the wire diameter d (x_1) and the number of active coils N (x_3).

Formally, the problem can be expressed as:

Minimize:

$$(N+2)Dd^2 \tag{37}$$

Subject to:

$$\begin{aligned}
g_1(\vec{x}) &= 1 - \frac{D^3 N}{71785 d^4} \leq 0 \\
g_2(\vec{x}) &= \frac{4D^2 - dD}{12566(Dd^3 - d^4)} + \frac{1}{5108 d^2} - 1 \leq 0 \\
g_3(\vec{x}) &= 1 - \frac{140.45 d}{D^2 N} \leq 0 \\
g_4(\vec{x}) &= \frac{D+d}{1.5} - 1 \leq 0
\end{aligned} \tag{38}$$

where $0.05 \leq x_1 \leq 2$, $0.25 \leq x_2 \leq 1.3$ y $2 \leq x_3 \leq 15$.

17. **Problem 17: (Design of a 10-bar plane truss)**

Consider the 10-bar plane truss shown in Figure 5 [10]. The problem is to find the moment of inertia of each member of this truss, such that we minimize its weight, subject to stress and displacement constraints. The weight of the truss is given by:

$$f(x) = \sum_{j=1}^{10} \rho \, A_j \, L_j \tag{39}$$

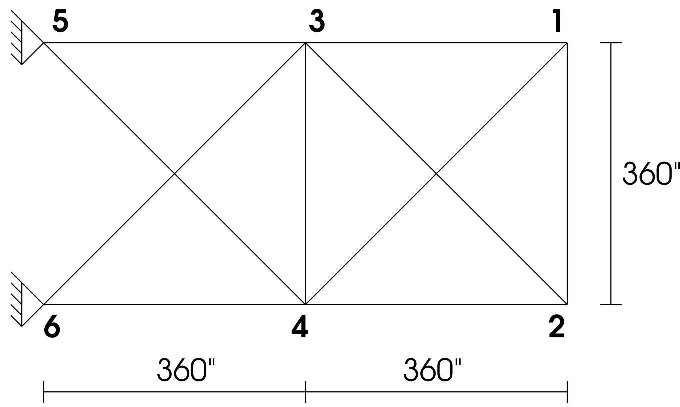

Figure 5: 10-bar plane truss used for problem 17.

where x is the candidate solution, A_j is the cross-sectional area of the jth member, L_j is the length of the jth member, and ρ is the weight density of the material.

The assumed data are: modulus of elasticity, $E = 1.0 \times 10^4$ ksi 68965.5 MPa), $\rho = 0.10$ lb/in^3 (2768.096 kg/m^3), and a load of 100 kips (45351.47 Kg) in the negative y-direction is applied at nodes 2 and 4. The maximum allowable stress of each member is called σ_a, and it is assumed to be ± 25 ksi (172.41 MPa). The maximum allowable displacement of each node (horizontal and vertical) is represented by u_a, and is assumed to be 2 inches (5.08 cm).

There are 10 stress constraints, and 12 displacement constraints (we can really assume only 8 displacement constraints because there are two nodes with zero displacement, but they will nevertheless be considered as additional constraints by the new approach). The moment of inertia of each element can be different, thus the problem has 10 design variables.

To get a measure of the difficulty of solving each of these problems, a ρ metric (as suggested by Koziel and Michalewicz [21]) was computed using the following expression:

$$\rho = |F|/|S| \tag{40}$$

where $|F|$ is the number of feasible solutions and $|S|$ is the total number of solutions randomly generated. In this work, $S = 1,000,000$ random solutions.
The different values of ρ for each of the functions chosen are shown in Table 1, where n is the number of decision variables, LI is the number of linear inequalities, NI the number of nonlinear inequalities, LE is the number of linear equalities and NE is the number of nonlinear equalities. It can be clearly seen that problems 5, 7 and 13 should

Problem	n	Type of function	ρ	LI	NI	LE	NE
1	13	quadratic	0.0003%	9	0	0	0
2	20	non linear	99.9973%	2	0	0	0
3	10	non linear	0.0026%	0	0	0	1
4	5	quadratic	27.0079%	4	2	0	0
5	4	non linear	0.0000%	2	0	0	3
6	2	non linear	0.0057%	0	2	0	0
7	10	quadratic	0.0000%	3	5	0	0
8	2	non linear	0.8581%	0	2	0	0
9	7	non linear	0.5199%	0	4	0	0
10	8	linear	0.0020%	6	0	0	0
11	2	quadratic	0.0973%	0	0	0	1
12	3	quadratic	4.7697%	0	9^3	0	0
13	5	non linear	0.0000%	0	0	1	2
14	4	quadratic	2.6859%	6	1	0	0
15	4	quadratic	39.6762%	3	1	0	0
16	3	quadratic	0.7537%	1	3	0	0
17	10	non linear	46.8070%	0	22	0	0

Table 1: Values of ρ for the 17 test problems chosen.

be the most difficult to solve since they present the lowest value of ρ.

We implemented five different types of ES:

- $(\mu + 1)$-ES
- $(\mu + \lambda)$-ES without correlated mutation.
- $(\mu + \lambda)$-ES with correlated mutation.
- (μ, λ)-ES without correlated mutation.
- (μ, λ)-ES with correlated mutation.

The number of fitness function evaluations was fixed to 350000 in all our experiments. We performed 30 runs for each problem and for each type of ES. Equality constraints were transformed into inequalities using a tolerance value of 0.0001 (see [13] for details of this transformation).

For the $(\mu + 1)$-ES the initial values are:

- $\sigma = 4.0$.
- $C = 0.99$.
- $\mu = 5$.
- Number of generations = 350000.

	$(\mu + 1)$-ES					
Problem	**Optimal**	**Best**	**Mean**	**Median**	**Worst**	**St. Dev.**
g01	-15.000000	-15.000000	-14.848614	-14.997996	-12.999997	0.410082
g02	0.803619	0.793083	0.698932	0.708804	0.576079	0.062927
g03	1.000000	1.000497	1.000486	1.000491	1.000424	0.000014
g04	-30665.539000	-30665.539062	-30665.441732	-30665.539062	-30663.496094	0.393918
*g05	5126.498000	1061.161621	3798.771277	3710.436401	7450.403320	1589.234278
g06	-6961.814000	-6961.813965	-6961.813965	-6961.813965	-6961.813965	0.000000
g07	24.306000	24.368050	24.702525	24.730650	25.516653	0.242956
g08	0.095825	0.095826	0.095826	0.095826	0.095826	0.000000
g09	680.630000	680.631653	680.673645	680.659271	680.915100	0.052483
*g10	7049.330700	5090.902832	11741.558219	11362.840820	19986.607422	3614.908836
g11	0.750000	0.749900	0.784395	0.776296	0.879522	0.037345
g12	1.000000	1.000000	1.000000	1.000000	1.000000	0.000000
g13	0.053950	0.060909	1.028332	0.929756	4.682147	0.852305
gpressure	6059.946000	6059.701660	6724.941455	6771.583984	7332.828613	460.417544
gbeam	1.728200	1.729834	1.782288	1.766287	1.881157	0.043994
gspring	0.012681	0.012679	0.013194	0.012849	0.015951	0.000820
gtruss10	5152.636000	5611.358887	6713.852327	6791.588379	7988.152832	613.365056

Table 2: Results obtained with the $(\mu + 1)$-ES in the 17 test problems with 350000 fitness function evaluations (The “*” indicates that no feasible solutions were found)

	Non-correlated $(\mu + \lambda)$-ES					
Problem	**Optimal**	**Best**	**Mean**	**Median**	**Worst**	**St. Dev.**
g01	-15.000000	-14.985728	-14.973915	-14.974497	-14.954204	0.007794
g02	0.803619	0.803607	0.800743	0.803503	0.792375	0.004637
g03	1.000000	0.473893	0.238810	0.242793	0.026602	0.113800
g04	-30665.539000	-30664.837891	-30651.001497	-30653.474609	-30619.619141	13.160883
*g05	5126.498000	5107.174316	5212.373470	5181.369629	5543.031250	102.461263
g06	-6961.814000	-6961.813965	-6938.453255	-6961.810791	-6567.754395	83.160125
g07	24.306000	24.328295	24.390978	24.392672	24.478491	0.046711
g08	0.095825	0.095826	0.095823	0.095826	0.095771	0.000010
g09	680.630000	680.630554	680.640236	680.636139	680.666443	0.010440
g10	7049.330700	7075.010254	7802.033024	7531.348877	10083.971680	762.989363
g11	0.750000	0.750572	0.882165	0.901714	0.998691	0.085372
g12	1.000000	1.000000	1.000000	1.000000	0.999997	0.000001
*g13	0.053950	0.984104	0.998943	0.999955	0.999999	0.003019
gpressure	6059.946000	6059.988281	6654.801432	6771.587646	7294.079590	298.294833
gbeam	1.728200	1.746999	2.033031	2.036841	2.450664	0.182135
gspring	0.012681	0.013091	0.015934	0.015670	0.020273	0.001891
gtruss10	5152.636000	5142.048340	5147.212077	5145.235840	5167.502441	6.303789

Table 3: Results obtained with the non-correlated $(\mu + \lambda)$-ES in the 17 test problems with 350000 fitness function evaluations (The “*” indicates that no feasible solutions were found)

For the $(\mu + \lambda)$-ES and (μ, λ)-ES panmictic discrete recombination for strategy parameters and decision variables was used. The learning rates values were calculated as shown in Section 2. The initial values for the standard deviations were 3.0 for all the decision variables.

The initial values for the remaining ES are:

- $\mu = 100$.
- $\lambda = 300$.
- Number of generations = 1166.

The results obtained for the $(\mu+1)$-ES are in Tables 2 and 7. For the non-correlated $(\mu + \lambda)$-ES and (μ, λ)-ES the information is on Tables 3 and 4. Finally, Tables 5 and 6 correspond to results of the correlated $(\mu + \lambda)$-ES and (μ, λ)-ES approaches.

	Non-orrelated (μ,λ)-ES					
Problem	Optimal	Best	Mean	Median	Worst	St. Dev.
g01	-15.000000	-14.994504	-14.971326	-14.975142	-14.931431	0.015573
g02	0.803619	0.792393	0.779795	0.784977	0.753796	0.011986
g03	1.000000	0.465430	0.165386	0.154534	0.007239	0.134065
g04	-30665.539000	-30432.130859	-30309.273307	-30297.312500	-30204.130859	52.561251
*g05	5126.498000	5041.400879	5162.947559	5157.134521	5336.575684	59.354507
g06	-6961.814000	-6916.589844	-6711.115853	-6789.253906	-6068.743164	206.012359
g07	24.306000	24.483683	24.928663	25.015615	25.484566	0.271122
g08	0.095825	0.095826	0.095826	0.095826	0.095821	0.000001
g09	680.630000	680.808533	681.351021	681.324066	682.871399	0.485906
g10	7049.330700	8024.879883	11721.520964	11677.316406	16982.537109	2319.203586
*g11	0.750000	0.783648	0.931193	0.940145	1.000796	0.053328
g12	1.000000	1.000000	1.000000	1.000000	1.000000	0.000000
*g13	0.053950	0.999126	0.999874	0.999993	1.000000	0.000240
gpressure	6059.946000	6470.276855	6909.340853	6943.404785	7417.326172	209.654662
gbeam	1.728200	2.329124	2.720000	2.699388	3.207800	0.213405
gspring	0.012681	0.014626	0.019007	0.018781	0.025735	0.002172
gtruss10	5152.636000	5153.757324	5356.443701	5373.206299	5696.909668	150.103553

Table 4: Results obtained with the non-correlated (μ, λ)-ES in the 17 test problems with 350000 fitness function evaluations (The “*” indicates that no feasible solutions were found)

	Correlated $(\mu+\lambda)$-ES					
Problem	Optimal	Best	Mean	Median	Worst	St. Dev.
g01	-15.000000	-14.999541	-14.997859	-14.998640	-14.973085	0.004617
g02	0.803619	0.803594	0.796618	0.792588	0.785246	0.005864
g03	1.000000	0.471707	0.202341	0.185342	0.085943	0.100457
g04	-30665.539000	-30665.529297	-30665.519661	-30665.519531	-30665.507812	0.005166
*g05	5126.498000	5125.168945	5233.366488	5163.475342	5697.309570	144.857450
g06	-6961.814000	-6961.760742	-6960.627539	-6960.971924	-6957.258789	1.145723
g07	24.306000	24.330238	24.422113	24.413397	24.563091	0.065209
g08	0.095825	0.095826	0.095826	0.095826	0.095826	0.000000
g09	680.630000	680.633423	680.638070	680.637848	680.644653	0.002704
g10	7049.330700	7294.707031	10857.807715	9929.192871	20743.082031	3355.115006
g11	0.750000	0.749904	0.752437	0.749950	0.812548	0.011315
g12	1.000000	1.000000	1.000000	1.000000	1.000000	0.000000
g13	0.053950	0.999998	1.000000	1.000000	1.000000	0.000000
gpressure	6059.946000	6090.513672	6661.626530	6771.584473	7332.829590	348.115280
gbeam	1.728200	1.725343	1.747797	1.737747	1.873749	0.032654
gspring	0.012681	0.012693	0.015069	0.014774	0.017993	0.001805
gtruss10	5152.636000	5146.706055	5323.578060	5364.495117	5526.920898	122.497753

Table 5: Results obtained with the Correlated $(\mu + \lambda)$-ES in the 17 test problems with 350000 fitness function evaluations (The “*” indicates that no feasible solutions were found)

	Correlated (μ,λ)-ES					
Problem	Optimal	Best	Mean	Median	Worst	St. Dev.
g01	-15.000000	-14.931046	-14.914536	-14.914993	-14.888850	0.009784
g02	0.803619	0.797201	0.777913	0.784871	0.748130	0.012513
g03	1.000000	0.445308	0.107894	0.040774	*0.000001	0.140491
g04	-30665.539000	-30664.216797	-30662.855143	-30662.590820	-30661.169922	0.771625
*g05	5126.498000	5121.693848	5150.308952	5138.650879	5266.957031	30.488439
g06	-6961.814000	-6802.235352	-6538.025928	-6541.951172	-6277.650879	127.244717
g07	24.306000	24.650963	24.886861	24.915041	25.238083	0.142073
g08	0.095825	0.095826	0.095822	0.095823	0.095811	0.000004
g09	680.630000	680.774780	681.138582	681.135864	681.498230	0.142602
g10	7049.330700	12146.522461	17457.792025	18413.143555	29076.019531	4163.691375
g11	0.750000	0.879374	0.952082	0.956904	0.997581	0.027962
g12	1.000000	1.000000	1.000000	1.000000	1.000000	0.000000
*g13	0.053950	0.999966	0.999996	0.999998	1.000000	0.000007
gpressure	6059.946000	6410.579102	7003.140755	7047.459961	7333.625000	289.044574
gbeam	1.728200	1.756485	1.777969	1.776924	1.817196	0.012992
gspring	0.012681	0.014593	0.017754	0.018169	0.018615	0.001093
gtruss10	5152.636000	5241.848633	5614.898079	5656.644775	5897.667480	161.051255

Table 6: Results obtained with the Correlated (μ, λ)-ES in the 17 test problems with 350000 fitness function evaluations (The “*” indicates that no feasible solutions were found)

	$(\mu + 1)$-ES					
Problem	**Optimal**	**Best**	**Mean**	**Median**	**Worst**	**St. Dev.**
g01	−15.000000	−15.000000	−14.915965	−14.993222	−14.578423	0.130738
g02	0.803619	0.794896	0.704833	0.727666	0.584765	0.064228
g03	1.000000	1.000497	1.000449	1.000488	0.999811	0.000141
g04	−30665.539000	−30665.539062	−30661.260612	−30665.539062	−30537.185547	23.040161
*g05	5126.498000	1243.262817	3780.308504	3537.221924	8152.663086	1896.802969
g06	−6961.814000	−6961.813965	−6961.813965	−6961.813965	−6961.813965	0.000000
g07	24.306000	24.362831	24.669716	24.697510	25.144272	0.195578
g08	0.095825	0.095826	0.095826	0.095826	0.095826	0.000000
g09	680.630000	680.631592	680.690181	680.665131	680.862244	0.069168
g10	7049.330700	*7098.121094	11695.400472	11155.280273	20063.314453	3732.664511
g11	0.750000	0.749900	0.783557	0.757834	0.892770	0.045494
g12	1.000000	1.000000	1.000000	1.000000	1.000000	0.000000
g13	0.053950	0.109717	1.480463	0.940179	20.088820	3.478791
gpressure	6059.946000	6059.701660	6644.890332	6590.829102	7332.828613	469.334608
gbeam	1.728200	1.727880	1.780659	1.758916	1.974485	0.061298
gspring	0.012681	0.012679	0.013414	0.013161	0.016459	0.000956
gtruss10	5152.636000	5516.977539	6620.212923	6587.030029	7373.917480	458.484033

Table 7: Results obtained with the $(\mu + 1)$-ES in the 17 test problems with 700000 fitness function evaluations (The "*" indicates that no feasible solutions were found)

5 Discussion of results

In order to allow a more reasonable discussion of results, we performed the following binary comparisons:

- Non-correlated $(\mu + \lambda)$-ES against correlated $(\mu + \lambda)$-ES.
- Non-correlated (μ, λ)-ES against correlated (μ, λ)-ES.
- "+" selection against "-" selection.
- Best overall approach in terms of offline performance.
- Best overall approach based on statistical measures.

5.1 Non-correlated $(\mu + \lambda)$-ES against correlated $(\mu + \lambda)$-ES

The results shown in Tables 3 and 5 show no evidence about the best overall performance (measured in terms of offline performance) of any of approaches implemented. The correlated version finds better results in problems 1, 4, 11, 15 and 16. Besides, this approach obtains a lower standard deviation in seven problems (1, 3, 4, 6, 9, 11 and 12). However the difference is not very significant.

5.2 Non-correlated (μ, λ)-ES against correlated (μ, λ)-ES

Tables 4 and 6 show that the correlated version works better, regarding the quality of the results, in only six problems (2, 4, 9, 14, 15 and 16). The standard deviation and the rest of the statistical values are better in ten test problems (1, 4, 5, 6, 7, 9, 11, 13, 15 and 16) for this correlated mutation version.

We argue that these results suggest that the correlated mutation does not improve the evolutionary search in constrained spaces in a significant way. This issue is important (computationally speaking), because there is an extra computational cost and storage associated with the implementation of this type of mutation. There is also evidence indicating that the comparison criteria explained in Section 3 added to the "," selection causes the search to be consistently trapped in local optimal solutions.

5.3 "+" selection against "-" selection

Analyzing the four types of ES implemented and their results from Tables 3 and 4 we can see that the implicit elitism of the "+" selection enhances the capacity of the ES search to find better results (even optimal results or very close like problems 1, 2, 4, 6, 7, 8, 9, 11, 12, 15 and problem 16 where the best known solutions are improved). Also, the "+" selection finds better results in all problems but in test function 1. Finally, the standard deviation values for the "," selection are better in problem 5 (where no feasible solutions were found), 8 (but there is no significant difference with respect to the "+" selection), 11, 12, 13 and 14.

Despite the fact that it is well known that the "," selection is less sensitive to get trapped in local optima [30, 6], in this experiment we can argue that elistism plays an important role in constrained optimization.

5.4 Best Overall Approach in Terms of Offline Performance

An unexpected result can be clearly seen in Table 2. The $(\mu+1)$-ES outperforms the remaining four approaches implemented. It finds the global optimum solution (or approximates it very well) in thirteen out of seventeen problems. The standad deviations in only problems 6, 8, 14, 15 and 16 are better than the best of the four remaining versions, the non-correlated $(\mu+\lambda)$-ES.

To analyze carefully this behavior, we performed another 30 runs with the $(\mu+1)$-ES but now using twice the number of fitness function evaluations adopted in our original experiments (700,000). See Table 7 for the new results.

This new experiment slightly improves the results only in five problems (2, 7, 9, 15 and 17). The statistical values are also better, but they are still not superior than those of the non-correlated $(\mu+\lambda)$-ES version.

5.5 Best Overall Approach based on Statistical Measures

In terms of average performance (based on our statistical measures), the best results were found by the non-correlated $(\mu+\lambda)$-ES. However, this approach was trapped in local optima in most of the problems.

5.6 Remarks

The last two points suggest that the use of a large number of strategy parameters difficults convergence in constrained search spaces. The use of only one sigma value for all the individuals during the evolutionary process seems to be enough to bias the search to the global feasible optimum solution or its neighborhood. Nonetheless, another mechanism is needed to improve the performance of the $(\mu+1)$-ES and its statistical measures.

It is also interesting to note that in test functions where the $(\mu + 1)$-ES could not find good results (2, 5, 10, 13 and 17), the $(\mu + \lambda)$-ES found better solutions (if not the optimum, either a better approximation to it or at least an almost-feasible solution). This could mean that this types of problems needs more of the explorative power of an evolutionary algorithm.

The current empirical study allows us to argue that a very simple ES approach, the $(\mu + 1)$-ES is enough to find competitive results in the seventeen test problems used in the benchmark provided to evaluate evolutionary algorithms in constrained search spaces. However, an additional mechanism must be added. It is also possible to have an ES with a moderated number of strategy parameters and without correlated mutation which may work reasonably well.

6 Conclusions and Future Work

An empirical study to analyze the usefulness of the natural self-adaptation mechanism of the Evolution Strategies was presented. We also explore the difference of using or not correlated mutation in ES adapted for constrained search spaces. Among the five different ES implemented, the most simple of them, the $(\mu + 1)$-ES, outperformed the other four in terms of the quality of the results found. The best statistical measures were obtained, however, by the non-correlated $(\mu + \lambda)$-ES.

The use of elitism was also remarked as an important factor to bias the ES to the feasible region of the search space and to find the optimum solution. Finally, it was empirically shown that the use of just one strategy parameter can lead the search to better solutions.

Our future work consists of:

- Suggesting a mechanism to improve the results obtained with the $(\mu + 1)$-ES (operators, short term memory or other than a Gaussian mutation operator).
- Exploring the use of a moderate number of strategy parameters in multimembered ESs to improve the results obtained.
- Modify the comparison criteria in order to get more diversity in the population.
- Incorporate a multiobjective-based mechanism to handle constraints [22].

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