Multiobjective Location of Automatic Voltage Regulators in a Radial Distribution Network Using a Micro Genetic Algorithm

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***Abstract*—In rural power systems, the automatic voltage regulators (AVRs) help to reduce energy losses and to improve the energy quality of electric utilities, compensating the voltage drops through distribution lines. In order to help electric companies in the decision-making process, this paper presents a method to define the optimal location of a set of AVRs in electric distribution networks. The optimization process is treated as a multiobjective problem considering the total power losses and the voltage drops in the system as the objectives to be optimized. A novel technique called micro genetic algorithm (μGA) is used to solve the multiobjective problem. This technique is capable of finding, in a very efficient way, the Pareto optimal solutions, giving the decision maker a set of possible (trade-off) solutions from which to choose.**

***Index Terms*—Evolutionary algorithms, losses, micro genetic algorithms, multiobjective optimization, optimization methods, power distribution, voltage control, voltage regulators.**

NOMENCLATURE

$\Delta V^j_{Regulator}$	Voltage variation of each regulator.
I_{VRj}	Current for jth AVR.
$\mathrm{I_{max}\ VR_j}$	Maximum current for jth AVR.
$\mathrm{i_j}$	Line current.
$\mathrm{L_T}$	Total power loss index.
NL, NB	Total lines and number of buses.
Rj	Series resistance of the distribution line.
$\mathrm{V_D}$	Deviation voltage index.
$\mathrm{V_K}$	Voltage for the kth node.
$V^j_{\min}, V^j_{\max}$	Minimum and maximum voltage values for each regulator.
$\mathrm{V_{ref}}$	Voltage reference.
t	Tap position of AVR.

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I. INTRODUCTION

KEEPING voltage profile within certain limits helps to reduce energy losses and improves voltage regulation. Voltage control is a difficult task because voltages are strongly influenced by dynamic load fluctuations. Therefore, utilities reinforce their power systems in order to have a better control over voltage variations. Improving a system's operations benefits both utilities and customers [1].

The operational voltage profile, in the design phase, can be improved by the use of analytical tools such as optimal power flow, voltage stability analysis, reliability analysis, etc. Moreover, it can be controlled by the installation of devices such as fixed and controlled capacitors banks, transformers with on-load tap changers (OLTCs), and automatic voltage regulators (AVRs) [2], [3]. However, the use of the AVR is constrained by its high investment cost. So, the optimal location of these devices becomes an important issue.

For many years, researchers have worked to define the optimal number, location, and sizing of capacitors banks to achieve voltage control while all operational constraints are satisfied, at different loading levels. Many single-objective optimization techniques have been applied to this problem, including heuristic methods such as expert systems, simulated annealing, and artificial neural networks [4]. Recently, fuzzy logic [1] and evolutionary algorithms [5]–[8] have also been used. In these cases, the objective function is defined by taking into account losses reduction, voltage constraints, and total cost.

Loss reduction and improvement of voltage profile have been also studied using OLTCs. Optimal power flow analysis is used to determine the optimal tap position and the ON/OFF state of the capacitor banks [9]. The same problem is solved in [10] using the losses equation as the objective function and voltage inequalities as constraints through the use of an artificial neural network. The works presented in [11] and [12] search the optimal location of OLTC and capacitor banks and also establish the optimal open/close state of sectionalizers in the system. In [13]–[15], the optimal number and location of AVRs are studied separately from the location and sizing of the capacitor banks problem. In this paper, the objective function used considers the peak power and energy losses. Finally, in the work of Safigianni and Salis in [16], the number and location of AVRs are determined by using a sequential algorithm. In addition to this, the objective function is defined by using the AVR's investment and maintenance costs and also the cost of the total energy losses.

In our work presented in [17], a method for optimal location of AVRs in radial distribution networks using a simple genetic algorithm was developed. In [17], the objective function was defined as a multiobjective problem (considering as objectives the total energy loss and the voltage deviation), but our solution approach adopted a weighted sum strategy, which is known to have some drawbacks [18]. The main results of this paper were the following: 1) genetic algorithms have a great potential to solve the problem of the optimal location of AVRs and 2) there is a reduced ensemble of solutions belonging to the trade-offs between the objectives for the proposed problem.

Summarizing, the optimal location of the capacitor banks problem has been widely studied. However, there are only a few publications that have treated the complex problem of the optimal location of AVRs in distribution networks, despite the fact that the benefits of including AVR devices are well known [19].

The method presented in this paper consists of determining the optimal location of the AVRs in the system, solving a multiobjective optimization problem that considers the simultaneous minimization of the active power losses and the voltage deviation, and taking into account the peak load demand.

This multiobjective problem is solved using the so-called micro genetic algorithm for multiobjective optimization, which is capable of finding nondominated solutions that represent the best possible trade-offs among the objectives. These nondominated solutions constitute the so-called Pareto optimal set. A Pareto optimal solution cannot be improved in any objective without worsening another one. The image of the Pareto optimal set (i.e., the objective function values corresponding to these nondominated solutions) is called the Pareto front. The decision maker (an engineer in our case) can choose any of the Pareto optimal solutions based on his/her own preferences. Normally, the Pareto front is used for that purpose, since the graphical representation of these solutions clearly indicates the sort of trade-offs achieved. One of the possible nondominated solutions can be preferred according to a particular criterion defined by the decision maker. In addition, the proposed method takes into account the rated power and tap constraints of AVR.

II. Problem Formulation

The optimization problem can be separated into two subproblems: 1) locating the AVR on the network and 2) the selection of the tap position for each of the AVRs. To solve these subproblems, we have considered that the system is well compensated regarding the reactive power reserves or demand and that this level of nodal compensation used in our model comes from a planning process, applied for a certain period of time.

A. Optimal Location of Voltage Regulators

The optimal location problem of an AVR is defined as a function of two objectives, one representing power losses reduction and the other one representing minimization of voltage deviations. Both are essential to ensure the security of power supply. It is important to note that the minimization of one of these objectives implies a decrease of the other one but not necessarily its minimization. It is difficult to formulate the problem in terms of cost incidence of these objectives over the system operation, because even when the cost incidence of power losses is clear, it is not the same for keeping the voltage values at the nodes close to the rated value [12].

The objective functions to be minimized are

$$\mathrm{L_T} = \sum_{j=1}^{NL} i_j^2 \cdot R_j \quad \text{(p.u.)} \tag{1}$$

$$\mathrm{V_D} = \sqrt{\sum_{k=1}^{NB} (V_k - V_{ref})^2} \quad \text{(p.u.)} \tag{2}$$

Subject to :

$$V_{\min}^j \leq \underset{\mathrm{Regulator}}{\Delta V^j} \leq V_{\max}^j \tag{3}$$

$$I_{VR\mathrm{j}} \leq \mathrm{I}_{maxVRj}. \tag{4}$$

Equations (3) and (4) represent the voltage range and the rated current of each AVR.

B. Selection of Tap Position

The determination of the tap position of each AVR is essential for solving the localization problem. In this kind of application, tap adjustment via successive displacement can force the solution to inadequate solutions or convergence problems. For this reason, a Newton–Raphson load flow algorithm is used to model, in a continuous way, the tap position as a state variable within the load flow calculations. This is more suitable for the optimization process and leads to better algorithmic performance.

C. Search Engine

Nowadays, there is a whole area of research, called "evolutionary multiobjective optimization" [18], where the inherent advantages of the evolutionary techniques are being used in order to find the Pareto optimal set of a problem. Opposite to conventional techniques, with an evolutionary algorithm, it is possible to find more than one element of the Pareto optimal set with a single run. In contrast, traditional mathematical programming techniques tend to generate Pareto optimal solutions one at a time. Additionally, evolutionary algorithms are less susceptible to the shape or continuity of the Pareto front, whereas these are serious concerns when adopting mathematical programming techniques [20].

Evolutionary multiobjective optimization techniques can be classified as follows [21]:

1) *Techniques not based on Pareto optimality*: linear and nonlinear aggregating methods [22], VEGA [23], lexicographic ordering [24], hybrids with the e-constraint method [25], etc.;
2) *Techniques based on Pareto optimality*: MOGA [26], NSGA [27], NPGA [28], μGA [29], PAES [30], NSGA-II [31], and SPEA2 [32], among others.

Considering the results of our former work [17], we know that in the AVR location problem, there is a reduced group of possible solutions (the Pareto optimal set) if we take into account the proposed objectives (losses reduction and minimization of voltage deviation). This is due to the correlation between the objectives and the discrete nature of the problem. Therefore, in this paper, we have chosen to give to the decision maker a set of Pareto optimal solutions, so that, with a practical criterion

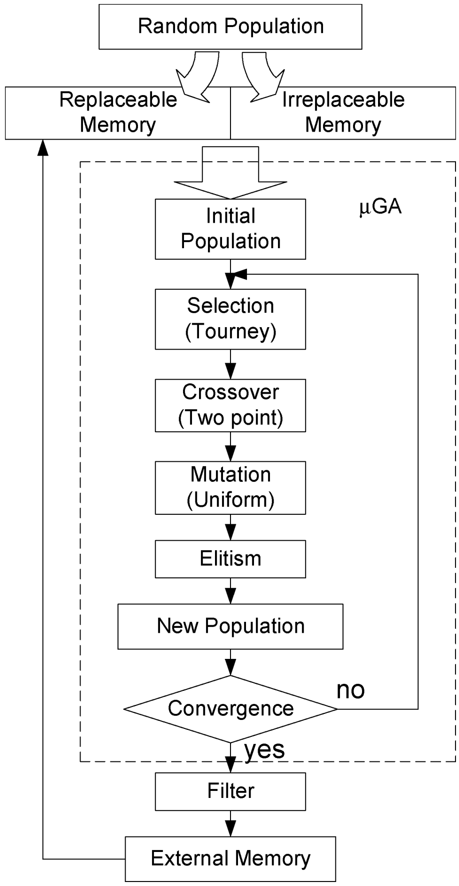

Fig. 1. Block diagram for the traditional μGA.

(based on experience), he/she can choose the most appropriate solution.

We decided to use the μGA proposed in [29] to solve our multiobjective optimization problem. This technique was developed by one of the co-authors of this paper, and its performance has been previously compared with respect to other techniques representative of the state-of-the-art in evolutionary multiobjective optimization in [29] and [33], giving excellent results.

D. Micro Genetic Algorithm

This technique improves the efficiency of the optimization process in comparison with other evolutionary algorithms because it applies the concept of Pareto dominance to a very small set of possible solutions (a maximum of five individuals are used in the main population) [29].

Fig. 1 illustrates the way in which the μGA works. First, a set of solutions is randomly generated. This random population (which is called population memory) is divided into two parts: a replaceable portion (RM) and an irreplaceable one (IRM). The irreplaceable portion never changes during the evolutionary search, since it constitutes the source of diversity of the approach. In contrast, the replaceable portion is modified after each cycle of the μGA.

The population of the μGA at the beginning of each of its cycles is taken (with a certain probability) from both portions of the population memory (i.e., the replaceable and the irreplaceable portions).

During each cycle, the μGA undergoes conventional operators from a simple genetic algorithm: tournament selection, two-point crossover, uniform mutation, and elitism. The μGA is run for a certain (predefined) number of iterations called "nominal convergence." After this, two nondominated solutions are chosen from the final population of the μGA, and they are compared with respect to the contents of the replaceable memory. Here, the aim is to replace two individuals from the replaceable memory that are dominated by these two solutions from the main population of the μGA. Over time, the replaceable memory will tend to have more nondominated solutions, some of which will be used in the initial populations of the μGA.

The original μGA also adopts an external archive where the nondominated solutions found along the evolutionary process are stored. This external archive is also used to spread the solutions found. Further details of the mechanisms of the μGA[1] can be found in [29].

III. Proposed Solution

A. Micro Genetic Algorithm Adapted to the AVR's Localization Problem

Considering that there is a reduced number of Pareto optimal solutions in the problem of our interest, we have removed the external memory of the process, which has considerably simplified the approach.

Instead of the external memory, we have adapted the replaceable memory to store our approximation of the Pareto front. Since the comparisons that take place (in the replaceable memory) involve the use of Pareto dominance, it is convenient to go over its formal definition (so far, this concept has only been informally introduced). The definition of Pareto dominance for two decision vectors $x, y \in \mathrm{F}$ (F refers to the feasible region) is shown next.

A vector $\vec{x} = (x_1, x_2, \ldots, x_k)$ is said to dominate (in a Pareto sense) another vector $\vec{y} = (y_1, y_2, \ldots, y_k)$ (denoted by $\vec{x} \underline{\prec} \vec{y}$) if and only if

$$\forall i \in (1, \ldots, k), x_i \leq y_i \wedge \exists i \in (1, \ldots, k) : x_i < y_i.$$

In other words, a vector dominates another one (in a Pareto sense) when it is less than or equal to (assuming minimization) with respect to all of its components, and it is strictly less with respect to at least one of them. Note that if a solution $\vec{x}$ does not dominate another solution $\vec{y}$, and $\vec{y}$ does not dominate $\vec{x}$, then both are nondominated with respect to each other (in other words, they are incomparable).

For example, if we consider the minimization of f_1 and f_2 and we have three vectors whose objective function values are the following: $A = (2, 1)$, $B = (3, 3)$, $C = (1, 2)$, we can say that A dominates $B (A \prec B)$ because $f_1(\mathrm{A}) < f_1(\mathrm{B})$ and $f_2(\mathrm{A}) < f_2(\mathrm{B})$. In the same way, we can say that C dominates $B (C \prec B)$ because $f_1(\mathrm{C}) < f_1(\mathrm{B})$ and $f_2(\mathrm{C}) < f_2(\mathrm{B})$. However, as $f_1(\mathrm{C}) < f_1(\mathrm{A})$ and $f_2(\mathrm{A}) < f_2(\mathrm{C})$, we can say that these two solutions do not dominate among them.

[1]The source code of the μGA for multiobjective optimization (in C/C++) is available for download at http://delta.cs.cinvestav.mx/~ccoello/EMOO/EMOOsoftware.html.

Whenever any of the solutions generated by the μGA dominates a solution (in a Pareto sense) in the replaceable memory, it replaces it.

Additionally, in this paper, we adopt a ranking system [34], which replaces the tournament selection originally adopted in the μGA. This ranking system aims to provide a higher selection probability to those elements that dominate more solutions.

B. Objective Function Evaluation

In order to evaluate the objective functions, the Newton–Raphson load flow algorithm is used. Here, the tap position of each AVR is considered as a state variable to be automatically adjusted within the iterative process [35]. Under this procedure, the tap position is considered as a continuous variable. However, there are no difficulties for including the tap as a discrete variable (see Appendix B).

The general formulation of the Newton–Raphson load flow algorithm is

$$\begin{bmatrix} H & N & D_p \\ M & L & D_q \end{bmatrix}^k \cdot \begin{bmatrix} \Delta\theta \\ \frac{\Delta V}{V} \\ \frac{\Delta t}{t} \end{bmatrix}^k = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^k. \tag{5}$$

$\mathrm{D_p}$ and $\mathrm{D_q}$ contain as many columns as voltage regulators, and their coefficients ($t\partial P/\partial t$ and $t\partial Q/\partial t$) are calculated using the pi-equivalent transformer model (the primary/secondary winding resistances and leakage reactances are considered). If one of the taps gets its maximum or minimum value, the regulated node becomes a PQ node unchanging the tap position. A constant power load model is used in this paper.

Once the load flow calculation is finished, (1) and (2) are used to evaluate the power losses and voltage deviation indexes.

C. Algorithm Description

The input data for the μGA for multiobjective optimization are the line parameters, the loads, the location and rated values of capacitors banks, and the number of voltage regulators to be installed.

In order to start the evolutionary multiobjective optimization process, the algorithm begins by giving a random population to the irreplaceable (or nonreplaceable) memory. Regarding that, most of the CPU time spent by the algorithm is associated with the objective function evaluations and the constraint verifications through the load flow, for which a database was provided. The aim is to store the candidates that already have been evaluated. Therefore, the objective function values are read from this memory if the procedure calls the candidates again to avoid repeated calculations.

The result of the algorithm is the Pareto region corresponding to the AVRs' location and tap position.

Fig. 2 shows the block diagram for the proposed method.

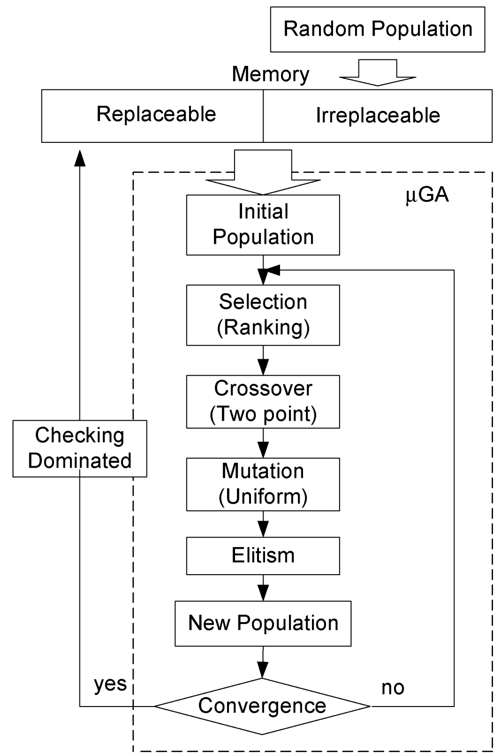

Fig. 2. Block diagram for the proposed μGA.

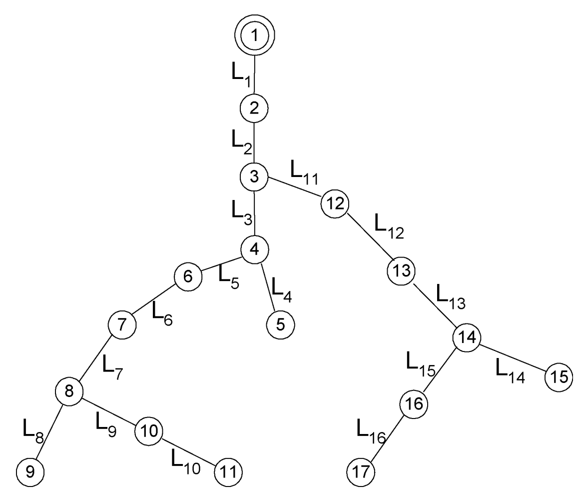

Fig. 3. Test system.

IV. APPLICATIONS

The algorithm proposed in this paper was developed in MATLAB, and the simulations were done on a computer with a Pentium IV processor, running at 1.6 GHz, and with 256 MB of RAM. A binary encoding was used in the μGA to represent the position of the AVR at the end of each line. The cycle's number and parameters for the μGA are specified according to the system's size. First, a test system is used in order to illustrate the way in which the proposed method works. Then, the real system (with a much higher complexity) is analyzed. All results of power loss and voltage deviation are expressed in per unit values.

Test System: The proposed method is applied to a radial test system with 16 lines and 17 nodes, shown in Fig. 3. A summary of the test system is shown in Appendix A. The power losses and voltage deviation values before installing the AVRs are 6.827×10^{-3} (p.u.) and 2.307×10^{-1} (p.u.), respectively.

Case 1 AVR: The parameters used to simulate this test system for the multiobjective μGA considering one AVR are shown in Table I (IRM in the table refers to the irreplaceable portion of

TABLE I
DATA FOR THE TEST SYSTEM: ONE AVR

Individuals of the IRM	:	5
Cycles of the process	:	1
Population size of the μGA	:	3
Generations of the μGA	:	3
% IRM and RM in the initial population memory	:	80/20

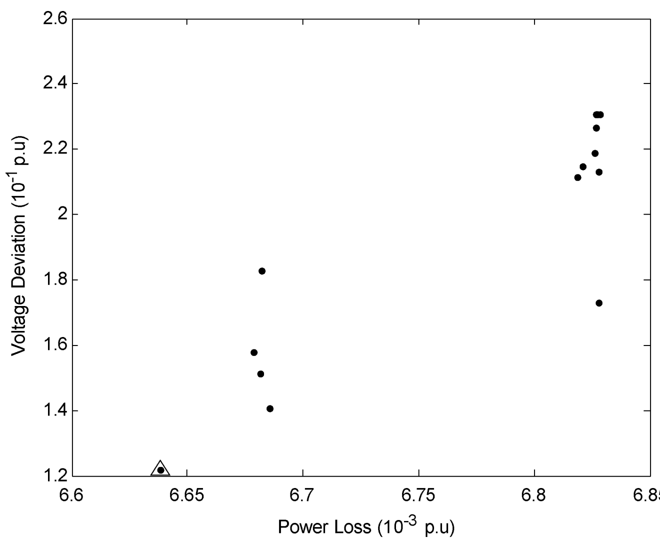

Fig. 4. Result for test system: one AVR.

TABLE II
RESULT TEST SYSTEM: ONE AVR

Location of AVR in line	:	2
Tap selected for AVR	:	0.9625
Power Loss ($x10^{-3}$) (p.u)	:	6.638
Voltage Deviation ($x10^{-1}$) (p.u)	:	1.216

TABLE III
DATA FOR THE TEST SYSTEM: TWO AVRS

Individuals of the IRM	:	10
Cycles of the process	:	8
Population size of the μGA	:	5
Generations of the μGA	:	3
% IRM and RM in the initial population memory	:	80/20

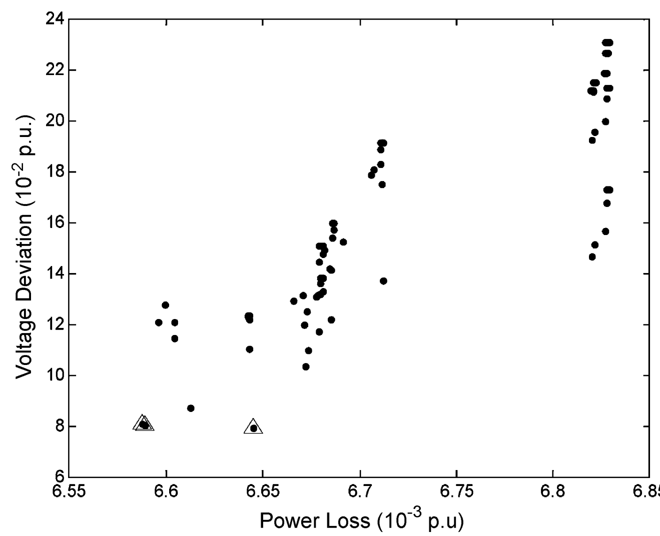

Fig. 5. Result for the test system: two AVRs.

TABLE IV
RESULT TEST SYSTEM: TWO AVRS

Solution		1	2	3
Location of AVR in line	:	2 and 6	2 and 5	2 and 7
Tap selected for AVR	:	t_2: 0.9629 t_6: 0.9726	t_2: 0.9629 t_5: 0.9753	t_2: 0.9629 t_7: 0.9456
Power Loss ($x10^{-3}$)	:	6.587	6.588	6.645
Voltage Deviation ($x10^{-2}$)	:	8.083	8.038	7.894

TABLE V
DATA FOR THE TEST SYSTEM: FOUR AVRS

Individuals of the IRM	:	100
Cycles of the process	:	20
Population size of the μGA	:	5
Generations of the μGA	:	3
% IRM and RM in the initial population memory	:	80/20

the population memory, whereas RM refers to the replaceable portion).

In Fig. 4, the results of the multiobjective process using only one AVR on the test system are compared in accordance to an exhaustive evaluation (EE) of all possible candidates (a triangle denotes a μGA solution, while a point denotes an EE solution). In this case, the Pareto region is formed only by one solution vector. These EE results match up with the solution found by the methodology proposed in this paper. The numerical results are shown in Table II.

The improvements of the power loss and voltage deviation functions are 2% and 45%, respectively.

The algorithm proposed is capable of finding the vector that dominates all the other solutions (in a Pareto sense) with only eight evaluations. This corresponds to the 50% of the evaluations required by the exhaustive process. The time involved in the simulation with the proposed algorithm was only 2.5 s, while with EE was 4.7 s.

Case 2 AVRs: In this case, the problem considers two AVRs. The parameters used in the μGA are shown in Table III.

In Fig. 5, the results of the proposed multiobjective process using two AVRs on the test system (marked with a triangle) are compared with the EE of all possible candidates (marked with a point). The numerical results are shown in Table IV. In this case, the Pareto region is formed by three vectors that are found by the proposed methodology performing 75 evaluations. In contrast, the entire exhaustive process required 120 evaluations. The time involved in the simulation with the proposed algorithm was 20 s, while with EE was 65 s. The ranges of improvement of the power loss and voltage deviation functions are 2.7%–3.5% and 65%–66%, respectively.

Case 4 AVRs: This problem considers four AVRs using the parameters shown in Table V.

In Fig. 6, the results of the multiobjective process using four AVRs on the test system (triangles) are compared with the EE of all possible candidates (points). In this case, the Pareto front

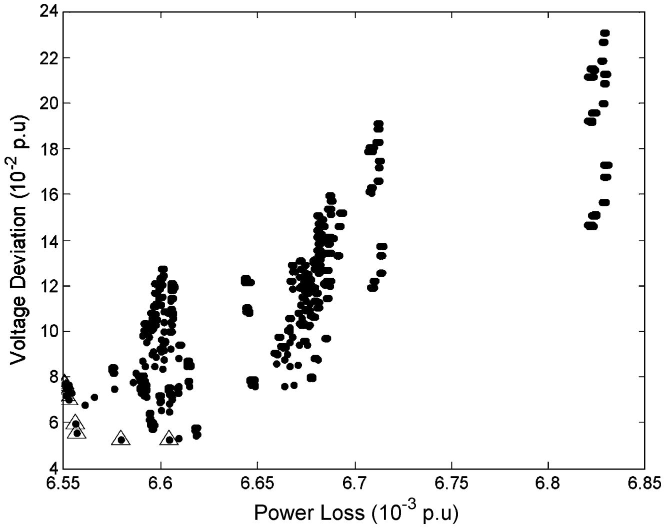

Fig. 6. Result for the test system: four AVRs.

TABLE VI
RESULT TEST SYSTEM: FOUR AVRS

Solution	Location of AVR	Power Loss ($x10^{-3}$)	Voltage Deviation ($x10^{-2}$)
1	2, 3, 6 and 7	6.604	5.241
2	1, 2, 6 and 16	6.550	7.725
3	1, 2, 5 and 9	6.552	7.011
4	1, 2, 5 and 7	6.556	5.533
5	1, 2, 6 and 9	6.551	7.168
6	1, 2, 6 and 7	6.555	5.981
7	1, 2, 6 and 15	6.550	7.651
8	1, 2, 3 and 7	6.579	5.259
9	1, 2, 6 and 12	6.550	7.491

consists of nine solutions. All of them are found by the proposed methodology without much difficulty. The most important numerical results are shown in Table VI. The ranges of improvement of the power loss and voltage deviation functions are 3.3%–4.0% and 67%–77%, respectively.

To find these results, the μGA performs only 576 evaluations, while the EE requires 1820 evaluations. This means that the algorithm is capable of finding all the possible solutions contained in the Pareto front evaluating only about 30% of the candidates. The time involved in the simulation with the proposed algorithm was 161 s, while EE was 713 s.

Real System: A real system of 229 nodes is analyzed. The system's one-line diagram, specifications, and results are detailed in [16]. The losses and voltage deviation values before installing the AVRs are 3.376×10^{-3} (p.u.) and 1.078 (p.u.) ,respectively. The base values used are 100 MVA and 20 kV. The method proposed in this reference gave, as a result, the location of one AVR at the end of the line 36 (node 37), solving a single-objective problem in which the energy losses reduction is considered as the only objective to be optimized.

Case 1 AVR: The parameters of this test system considering one AVR are shown in Table VII.

In Fig. 7, the results of the multiobjective process are compared to the EE for all possible candidates. Here, a triangle denotes an μGA solution, while a point denotes an EE solution. In this case, the Pareto region has six solution vectors. These EE results match with those found by the proposed methodology. The numerical results are shown in Table VIII.

TABLE VII
DATA FOR THE REAL SYSTEM: ONE AVR

Individuals of the IRM	:	100
Cycles of the process	:	30
Population size of the µGA	:	5
Generations of the µGA	:	3
% IRM and RM in the initial population memory	:	80/20

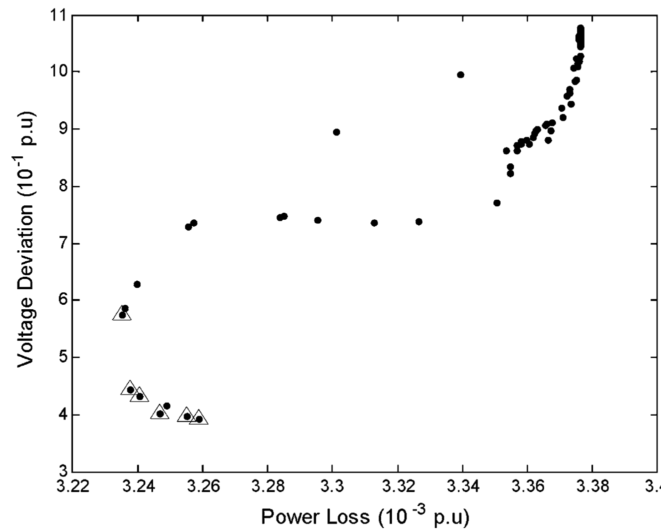

Fig. 7. Result for the real system: one AVR.

TABLE VIII
RESULT FOR THE REAL SYSTEM: ONE AVR

Solution		1	2	3	4	5	6
Location of AVR	:	13	15	21	23	34	36
Tap selected for AVR	:	0.963	0.953	0.951	0.948	0.944	0.943
Power Loss ($x10^{-3}$)	:	3.234	3.237	3.240	3.246	3.255	3.258
Voltage Deviation ($x10^{-1}$)	:	5.757	4.444	4.333	4.015	3.967	3.924

To find these results, the μGA needs to evaluate 140 individuals, while the EE requires 228 evaluations. This means that, evaluating about 60% of the candidates, the developed algorithm is capable to generate the entire Pareto optimal set of the problem. The time involved in the simulation with the proposed algorithm was 4820 s, while EE required 6720 s. The ranges of improvement of the power loss and voltage deviation functions are 3.5%–4.2% and 47%–64%, respectively.

Case 3 AVRs: This problem considers three AVRs using the parameters shown in Table IX. In this case, it is very difficult to perform the EE to compare the results given by the proposed model; for this reason, this procedure was not used (more than two million evaluations would be needed with an estimated CPU time of two years). For this reason, Fig. 8 only shows the solutions of the μGA. The time taken for the simulation was 28 h.

The most important numerical results are shown in Table X, in which the 13 Pareto optimal solutions found by the multiobjective optimization process are shown. The final locations of regulators are concentrated in a set of four solutions. The ranges

TABLE IX
DATA FOR THE REAL SYSTEM: THREE AVR

Individuals of the IRM	:	500
Cycles of the process	:	350
Population size of the µGA	:	5
Generations of the µGA	:	3
% IRM and RM in the initial population memory	:	80/20

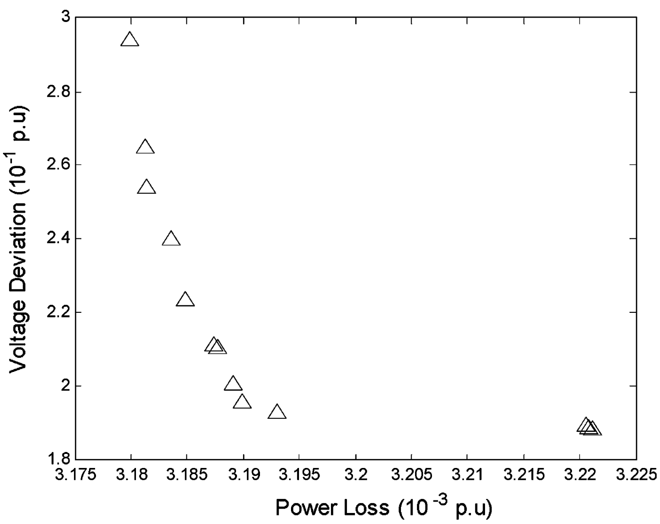

Fig. 8. Result for the real system: three AVR.

TABLE X
RESULT FOR THE REAL SYSTEM: THREE AVRS

Solution	Location of AVR	Power Loss ($x10^{-3}$)	Voltage Deviation ($x10^{-1}$)
1	4, 15 and 142	3.181	2.645
2	4, 21 and 142	3.181	2.538
3	4, 15 and 140	3.179	2.938
4	4, 34 and 144	3.187	2.102
5	6, 34 and 144	3.187	2.111
6	6, 32 and 144	3.184	2.234
7	6, 32 and 142	3.183	2.396
8	9, 34 and 144	3.189	2.004
9	9, 36 and 144	3.189	1.957
10	9, 34 and 154	3.189	2.004
11	15, 49 and 144	3.220	1.891
12	15, 62 and 144	3.221	1.882
13	15, 57 and 144	3.220	1.887

of improvement of the power loss and voltage deviation functions are 4.6%–5.8% and 73%–83%, respectively. Twelve thousand evaluations were performed to find the Pareto front of this problem.

V. CONCLUSION

In this paper, the optimal location of AVRs in a power distribution system is studied. The multiobjective problem is tackled using a novel technique based on genetic algorithms called the micro genetic algorithm for multiobjective optimization. This technique searches Pareto optimal solutions using a very small population size and a set of special operators. The multiobjective problem was formulated taking into account two objectives to be minimized: the total power losses and the system's voltage deviation. Constraints such as the maximal deviation of tap position and the standardized nominal values of AVRs are considered. To avoid numerical convergence problems, the tap position of each AVR is treated as a state variable in the Newton–Raphson load flow algorithm. We found that the μGA is well suited to solve this combinatorial optimization problem. Our empirical study indicated that the μGA is able to generate a set of good trade-off solutions in a reasonable CPU time.

The method's performance and hypothesis are evaluated with a simple 17-node test and with a 229-node real system. In both cases, the procedure was compared with the EE procedure (only in the one AVR case for the real-world system). When analyzing the CPU time required by each simulation in more detail, we realized that over 80% of the total CPU time is spent in the load flow algorithm. Thus, it is evident that if the computer code used for the load flow is optimized and a compilable programming language (e.g., C or C++) is used instead of MATLAB, a considerable speedup in the total execution time may be achieved. The positive performance of the proposed method is verified. This gives the decision maker important information to perform the optimal location of the AVRs in the system, regarding the localization of the AVRs.

TABLE XI
TEST SYSTEM LINE AND LOADS DATA

in	out	R(p.u)	X(p.u)	MW	Mvar
1	2	0.05	0.05	0.8	0.6
2	3	0.11	0.11	0.8	0.6
3	4	0.15	0.11	0.8	0.6
4	5	0.08	0.11	0.8	0.64
4	6	0.11	0.11	1.2	0.16
6	7	0.04	0.04	0.8	-0.16
7	8	0.80	0.11	0.6	0.48
8	9	0.075	0.10	1.6	1.08
8	10	0.09	0.18	2.0	0.72
10	11	0.04	0.04	0.4	0.36
3	12	0.11	0.11	0.24	-0.20
12	13	0.04	0.04	1.8	0.80
13	14	0.09	0.12	0.4	0.36
14	15	0.11	0.11	0.4	-0.44
14	16	0.08	0.11	0.4	0.36
16	17	0.04	0.04	0.84	-0.32

APPENDIX A
DATA OF TEST SYSTEMS

The rated value for the AVRs are: 5, 10, and 15 MVA with a tap variation range of $\pm 10\%$, with tap increments of 0.625%. A summary of the test system is shown in Table XI; the base values used are 100 MVA and 23 kV.

APPENDIX B
RESULT WITH STANDARDIZED TAPS

The aim of this Appendix is to show the results of the AVR location, considering the tap as a discrete variable. These results were obtained by adopting a standardization process in the load flow [36].

The results for the location of one AVR in the real system are shown in Table XII and Fig. 9 (a triangle denotes a μGA solution, while a point denotes an EE solution). In this case, the Pareto region has only three solution vectors. They enclose the medium values of the objective function and the tap positions,

TABLE XII
RESULT FOR THE REAL SYSTEM: ONE AVR

Solution		1	2	3
Location of AVR in line	:	11	21	34
Tap selected for AVR	:	0.9625	0.95	0.94375
Power Loss ($x10^{-3}$)	:	3.227	3.236	3.253
Voltage Deviation ($x10^{-1}$)	:	5.593	4.147	3.874

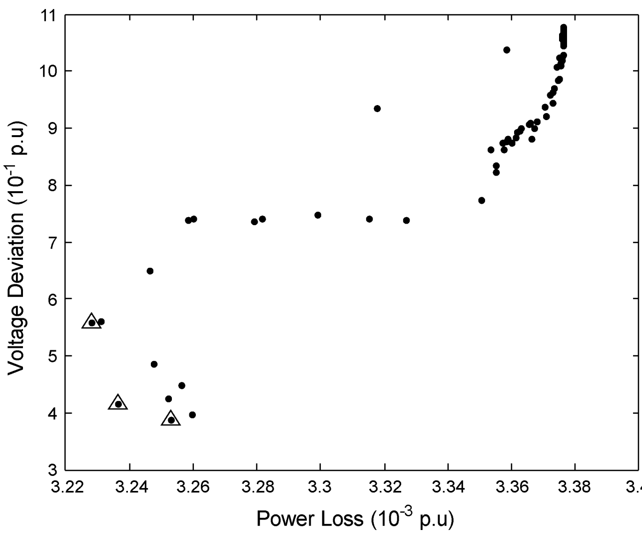

Fig. 9. Result for the real system: one AVR.

regarding the solutions considering the tap as a continuous variable (see Table VIII). Besides, when reducing the number of alternatives (points, if we refer to the Pareto region), the selection of one solution becomes easier.

The time involved in the simulation was 8147 s. The increase in CPU time required by the simulation is due to the standardization process, the one that requires a bigger calculations number and additional iterations to reach convergence.

APPENDIX C
GLOSSARY

Nondominated solutions: A nondominated solution represents the best possible trade-off among the objectives.

Pareto front: The objective function values corresponding to these nondominated solutions are called the Pareto front.

Ranking System: Method used to select individuals from the population, based on the concept of Pareto dominance (nondominated solutions are given a higher rank and, therefore, a higher probability of being selected).

VEGA: vector evaluated genetic algorithm
MOGA: multiobjective genetic algorithm
NSGA: nondominated sorting genetic algorithm
NPGA: niched Pareto genetic algorithm
PAES: Pareto archived evolution strategy
NSGA-II: nondominated sorting genetic algorithm-II

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