Ranking Methods in Many-objective Evolutionary Algorithms

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Abstract

This chapter presents a comparative study of different ranking methods on many-objective problems. The aim of this work is to investigate the effectiveness of different approaches in order to determine any possible limitations and/or advantages of each of the ranking methods studied and, in general, their performance. Thus, the results may help practitioners to select a suitable ranking method for a problem at hand, and can serve researchers as a guideline to develop new ranking schemes or further extensions of the Pareto optimality relation.

1 Introduction

In many disciplines, optimization problems have two or more objectives, which are normally in conflict with one another, and that we wish to optimize simultaneously. These are called *multi-objective optimization problems* (MOPs), and their solution involves the design of algorithms different from those adopted for dealing with single-objective optimization problems. In single-objective optimization, the determination of the optimum among a set of given solutions is clear. However, in the absence of preference information, in multi-objective optimization there does not exist a unique or straightforward way to determine if a solution is better than other. The notion of optimality most commonly adopted is the one called *Pareto optimality* [27] which leads to trade-offs among the objectives. Thus, by using this

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relation, it is not possible to obtain a single solution, but instead, we produce a set of them called the *Pareto optimal set*.

Nowadays, Multi-objective Evolutionary Algorithms (MOEAs) have shown an acceptable performance in many real-world problems with their origins in engineering, scientific and industrial areas [4]. Nonetheless, most of the publications in this area consider problems with only two or three objectives, in spite of the fact that many real-world problems involve a larger number of objectives (4 or more).[1] The MOEAs that are based on traditional Pareto dominance and that are the most representative and cited in the current literature are: PAES [20], NSGA-II [7], SPEA2 [39] and micro-GA [5]. Besides the difficulty to analyze the Pareto front when there are more than three objectives, recent studies [17, 18, 28, 36] have shown that MOEAs based on Pareto optimality have difficulties to find a good Pareto front approximation in problems with a large number of objectives, which are called *many-objective problems*.[2] One of the reasons for this limitation is that the proportion of nondominated solutions (*i.e.*, equally good solutions regarding Pareto optimality) in a population increases rapidly with the number of objectives. In [14] it is shown that this number goes to infinity when the number of objectives approaches infinity. This implies that in the presence of a large number of objectives the selection of new solutions is carried out almost at random since a large number of the solutions are equally good.

In the current literature we can identify two approaches commonly adopted to cope with many-objective problems, namely: *i*) to propose relaxed forms of Pareto optimality as in [2, 11, 14, 33], and *ii*) to reduce the number of objectives of the problem to ease the decision making or the search processes [3, 8, 21].

Since relaxed forms of Pareto optimality are the most common approach found in literature, in this chapter we present a comparative study that tries to reveal the advantages and disadvantages of some ranking methods used as optimality relations in many-objective problems. That is, methods that induce a partial order in a set of vectors but with a finer grain resolution than the one induced by traditional Pareto optimality. The assessment to the different ranking methods is based on the distribution of the ranks (*i.e.,* the number of ranks and the number of solutions in each rank) and on the plots of the solutions in decision space versus their ranks. The ranking methods considered in this study include redefinitions of the Pareto optimality relation or methods that complement it by inducing a finer ordering on the nondominated solutions found. In Section 5, we will briefly describe the ranking methods considered for this study. These methods were adopted because they follow considerably different approaches, do not require extra parameters and have shown promising results.

The remainder of this chapter is organized as follows. In Section 2, we provide some basic concepts related to multi-objective optimization. We mention the scala-

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[1] See for example [23, 29] in which the use of MOEAs in circuit optimization is discussed.

[2] Although this term is commonly used in the specialized literature, there is no consensus about how many objectives are considered 'many'. However, judging by the scalability difficulties shown by Pareto-based MOEAs, we consider that we deal with a many-objective problem if it has more than 4 objectives.

bility problems of the Pareto optimality relation in Section 3 and the optimality relations are discussed in Section 4. The ranking methods are all discussed in Sections 5 and 6. Section 7 includes the comparison and analysis of the ranking methods. Finally, in Section 8, we provide our final remarks with respect to the comparative study performed.

2 Background Concepts

2.1 Multi-objective optimization problem (MOP)

The Multi-objective optimization problem can be formally defined as the problem of finding:

$\mathbf{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ which satisfies the m inequality constraints:

$$g_i(\mathbf{x}) \leq 0; i = 1, \ldots, m$$

the p equality constraints:

$$h_i(\mathbf{x}) = 0; i = 1, \ldots, p$$

and optimizes the vector function:

$$\mathbf{f}(\mathbf{x}) = f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})$$

In other words, we aim to determine from among the set $\mathscr{F}$ of all vectors (points) which satisfy the constraints those that yield the optimum values for all the k objective functions simultaneously. The constraints define the feasible region $\mathscr{F}$ and any point $\mathbf{x}$ in the feasible region is called a feasible point.

2.2 Pareto Dominance

Pareto Dominance is formally defined as follows:

A vector $\mathbf{u} = (u_1, \ldots, u_k)$ *is said to dominate* $\mathbf{v} = (v_1, \ldots, v_k)$ *if and only if* $\mathbf{u}$ *is partially less than* $\mathbf{v}$*, i.e.,* $\forall i \in (1, \ldots, k), u_i \leq v_i \wedge \exists i \in (1, \ldots, k) : u_i < v_i$ (assuming minimization).

In order to say that a solution dominates another one, this one needs to be strictly better in at least one objective, and not worse in any of them. So when we are comparing two different solutions A and B, there are 3 possibilities:

- A dominates B
- A is dominated by B
- A and B are nondominated

2.3 Pareto Optimality

The formal definition of *Pareto optimality* is provided next:

A solution $\mathbf{x_u} \in \mathscr{F}$ *(where* $\mathscr{F}$ *is the feasible region) is said to be Pareto optimal if and only if there is no* $\mathbf{x_v} \in \mathscr{F}$ *for which* $\mathbf{v} = f(\mathbf{x_v}) = (v_1, \ldots, v_k)$ *dominates* $\mathbf{u} = f(\mathbf{x_u}) = (u_1, \ldots, u_k)$, where k is the number of objectives.

In other words, this definition says that $\mathbf{x}_u$ is Pareto optimal if there exists no feasible vector $\mathbf{x_v}$ which would decrease some objective without causing a simultaneous increase in at least one other objective (assuming minimization).

This definition does not provide us a single solution (in decision variable space), but a set of solutions which form the so-called *Pareto Optimal Set* (P^*). The vectors that correspond to the solutions included in the Pareto optimal set are *nondominated*.

2.4 Pareto Front

When all the nondominated solutions of a MOP are plotted in objective function space, their nondominated vectors are collectively known as the *Pareto Front* (PF^*). Formally:

$$PF^* := \{\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})) | \mathbf{x} \in P^*\}$$

It is, in general, impossible to find an analytical expression that defines the Pareto front of a problem, so the most common way to obtain the Pareto front is to compute a sufficient number of points in the feasible region, and then filter out the nondominated vectors from them.

The previous definitions are graphically depicted in Figure 1, showing the *Pareto front*, the *Pareto Optimal Set* and the *dominance* relations among solutions. Please refer to [4] for more in-depth information about multi-objective optimization.

3 Scalability Problems when Dealing with Many Objectives

Since the implementation of the first MOEA in the mid-1980s [31], a wide variety of new MOEAs have been proposed, gradually improving in effectiveness and efficiency for solving MOPs. However, the typical validation of such MOEAs is done by adopting test problems with only two or three objectives, and soon researchers realized that the traditional Pareto ranking schemes (in spread use today) scale poorly when the number of objectives increases. It is therefore, a natural step to start designing MOEAs that can deal with problems having a large number of objectives, and therefore the importance of studies such as the one presented in this chapter.

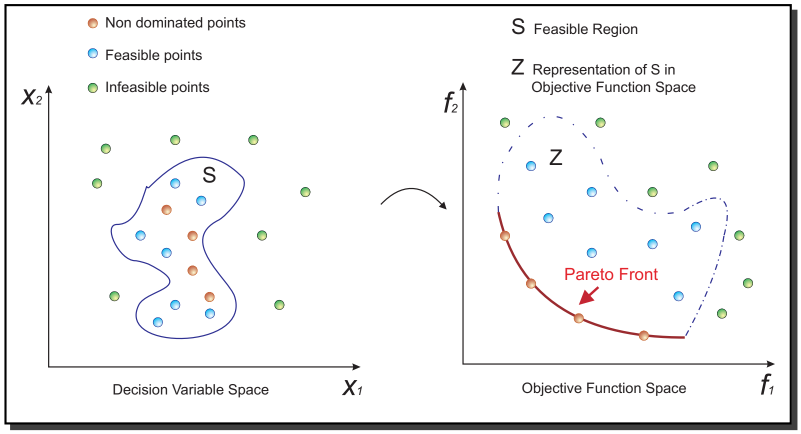

Fig. 1 Mapping of the Pareto Optimal Solutions to the Objective Function Space.

Recent experimental [17, 18, 28, 36] and analytical [6, 19, 34] studies have shown that MOEAs based on Pareto optimality scale poorly in MOPs with a high number of objectives (4 or more). Although this limitation seems to affect only the Pareto-based MOEAs, optimization problems with a large number of objectives (also known as many-objective problems) introduce some difficulties common to any other multi-objective optimizer. Three of the most serious difficulties due to high dimensionality are the following:

1. *Deterioration of the Search Ability*. One of the reasons for this problem is that the proportion of nondominated solutions (*i.e.*, equally good solutions) in a population increases rapidly with the number of objectives [14]. According to Bentley *et al*. [1] the number of nondominated k-dimensional vectors on a set of size n is $O(\ln^{k-1} n)$. This implies that in problems with a large number objectives, the selection of solutions is carried out almost at random or guided by diversity criteria. In fact, Mostaghim and Schmeck [25] have shown that a random search optimizer achieves better results than NSGA-II [7] in a problem with 10 objectives.
2. *Dimensionality of the Pareto front*. Due to the 'curse of dimensionality' the number of points required to represent accurately a Pareto front increases exponentially with the number of objectives. The number of points necessary to represent a k-dimensional Pareto front with resolution r is given by $O(kr^{k-1})$ (*e.g.*, see [32]). This poses a challenge both to the data structures to efficiently manage that number of points and to the density estimators to achieve an even distribution of the solutions along the Pareto front.
3. *Visualization of the Pareto front*. Clearly, with more than three objectives is not possible to plot the Pareto front as usual. This is a serious problem since visualization plays a key role for a proper decision making. Parallel coordinates [38] and self-organizing maps [26] are some of the methods proposed to ease the

decision making in many-objective problems. However, more research in this area is required.

Currently, there are mainly two approaches to solve many-objective problems, namely:

1. Adopt or propose an optimality relation that yields a solution ordering finer than that yielded by Pareto optimality. Among these alternative relations we can find average ranking [2], *k*-optimality [14], preference order ranking [11], favour relation [33], and a method that controls the dominance area [30]. Besides providing a richer ordering of the solutions, these relations obtain an optimal set that usually is a subset of the Pareto optimal set. Therefore, these techniques can be used as a remedy for the first and second issues of the previous enumeration.
2. Reduce the number of objectives of the problem during the search process or, *a posteriori*, during the decision making process [3, 8, 21]. The main goal of this kind of reduction techniques is to identify the redundant objectives (or redundant to some degree) in order to discard them. A redundant objective is one that can be removed without changing the dominance relation[3] induced by the original objective set.

4 Optimality Relations to Discriminate Solutions

Optimization techniques search a problem domain for the most efficient solution. Some techniques focus on one solution at a time and some other techniques can process a set of solutions (called "population"). The optimizer locates a single point in the objective space, tests it, compares its fitness to the previous best result, and determines the next point to test. Population-based optimizers are more complex, because they need to identify a whole set of points in the objective space, test all the points, rank their fitnesses, and determine the next set of solutions to test.

An important aspect of population-based algorithms is their need to rank the solutions before they are processed by the optimizer. The ranking procedure takes place during the evaluation process, after the objective functions have been evaluated and an array filled with result values has been created. Ranking means to transform the resultant array that is produced by the multi-objective evaluation into a resultant vector. Ranking is required because a set of solutions in the problem domain is being tested and the results have to be presented to the optimization paradigm in a uniform structure. Ranking is fundamental to those methods because it guides the search: the best solutions in a set are given the top ranking. The optimizer relies on the top ranks to identify high-potential regions of the search space to be explored. Therefore, any ranking method that is used needs to be efficient. The ranking method has to be robust enough to handle multiple objectives (i.e., it must be scalable). If the ranking method adopted is not robust, then many different ranking methods will have

[3] The dominance relation induced by a given set $\mathscr{F}$ of objectives is defined by $\preceq_{\mathscr{F}} = \{(\mathbf{x},\mathbf{y}) | \forall f_i \in \mathscr{F} : f_i(\mathbf{x}) \leq f_i(\mathbf{y})\}$.

to be used as the number of objectives increases. To develop a generalized ranking method the following question has to be answered: Can a single ranking method be developed that remains consistent across a range of many objectives?

The taxonomy of approaches that we will cover in this comparative study is shown in Figure 2. These techniques were selected because they were proposed specially for many-objective optimization problems and they represent a substantial difference with respect to the Pareto optimality relation. In the literature, we can find other optimality relations. However, they are minor variations, whether it be of Pareto optimality or some other optimality relation studied here. For instance, the fuzzy optimality relation presented in [37] is a variant of that defined by Farina and Amato [15] and the winning score relation [22] is equivalent to the average ranking method.

In this proposed taxonomy, we divided the ranking techniques in two groups: (1) those that do not need extra parameters to rank all the solutions, and (2) those in which at least one parameter is required to rank the solutions properly. Each of these two groups are discussed in this chapter.

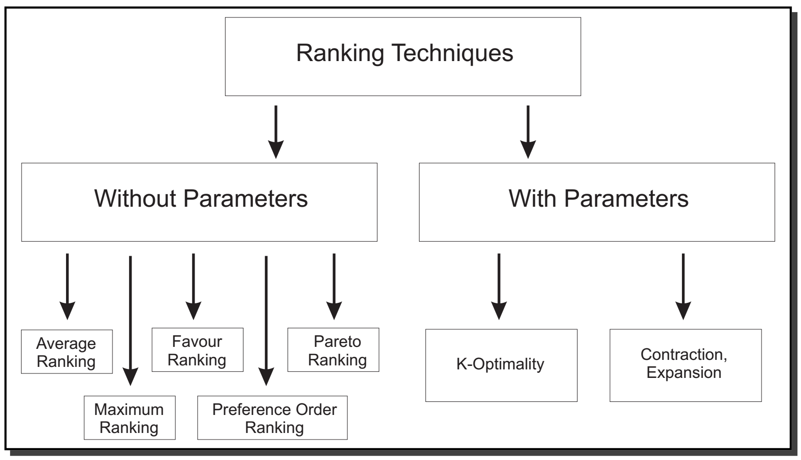

Fig. 2 Mapping of the Pareto Optimal Solutions to the Objective Function Space.

5 Ranking Methods without Parameters

5.1 Average and Maximum Ranking Methods

Although without a specific interest in many-objective problems, Bentley and Wakefield [2] proposed three alternative ranking methods to Pareto optimality, namely: average ranking (AR), sum of ratios (SR) and maximum ranking (MR). The AR method computes for each solution a different rank considering each objective independently. The final rank of a solution is obtained by summing all their ranks on each objective. Table 1 illustrates the AR method with a small example with six 3-objective solutions.

Table 1 An example of the Average, Maximum, Favour and Preference Order ranking methods.

Solution	rank 1	rank 2	rank 3	a) Average	b) Maximum	c) Favour	d) Preference Order
(4, 3, 5)	3	3	4	10	3	3	3
(1, 4, 7)	1	4	6	11	1	3	2
(6, 2, 2)	4	2	1	7	1	1	1
(7, 7, 6)	5	5	5	15	5	4	5
(8, 1, 3)	6	1	2	9	1	2	3
(3, 8, 4)	2	6	3	11	2	3	4

In this study, an equivalent method is used to compute the *AR* for each solution. First, the following matrix is defined for solutions $\mathbf{x}_i$ and $\mathbf{x}_j$:

$$a_{ijk} = \begin{cases} 1 \text{ if } f_k(\mathbf{x}_i) < f_k(\mathbf{x}_j) \\ 0 \text{ if } f_k(\mathbf{x}_i) = f_k(\mathbf{x}_j) \\ -1 \text{ if } f_k(\mathbf{x}_i) > f_k(\mathbf{x}_j) \end{cases}$$

From these values, the rank *AR* for each solution $\mathbf{x}_i$ is computed by:

$$AR(\mathbf{x}_i) = KN - \sum_{k=1}^{K} \sum_{j \neq i}^{N} a_{ijk},$$

where K is the number of objectives, N the number of solutions.

The maximum ranking takes the best rank as the global rank for each solution. Clearly this method favors extreme solutions, i.e., solutions with high performance in some of the objectives, although with poor overall performance. Table 1 shows an example of the use of this method.

5.2 Favour Ranking

This ranking method was proposed by Drechsler in [13] and consists of a new relation called *favour*. This technique requires no user interaction and can handle infeasible solutions.

$$x <_f y \Leftrightarrow |i : f_i(x) < f_i(y), 1 \leq i \leq n| > |j : f_j(x) < f_j(y), 1 \leq j \leq n| \quad (1)$$

This means that x is *favoured* to y ($x <_f y$) iff i components of x are better than the corresponding components of y and only j components of y are better than the corresponding components of x. For example: $f_{x_1} = (1,1,2)$ and $f_{x_2} = (5,3,1)$, then we have that: $f_{x_1} <_f f_{x_2}$.

Also, in this model, the authors proposed that the solutions are divided into so-called *Satisfiability Classes* (SCs) depending on their quality. Solutions of the same quality belong to the same SC. This property helps the mechanism in using a graph representation to describe properly the relation *favour* ($<_f$) , in which each element is a node and preferences are given by edges. The relation $<_f$ is not transitive, thus the relation $<_f$ can generate "cycles" in the graph, causing elements that describe a cycle to be denoted as not comparable and they are included in the same SC given the same rank to each of the elements in that cycle. The graph that contains all the populations (G_Z) needs to be a directed graph without internal cycles, so all the cycles have to be identified and replaced by a single node representing the single cycle.

Once we have the final directed graph (G_Z), we know that (G_Z) is acyclic, and it is possible to determine a *level sorting* of the nodes. For each node in G_Z we define a SC. Level sorting of the nodes in G_Z determines the ranking of the SCs; each level contains at least one node of G_Z. Then each level corresponds to exactly one SC. Using the level sorting, it is possible to group nodes (sets of solutions) that are not connected by an edge in the same SC. These solutions are not comparable with respect to relation $<_f$ and thus they should be ranked in the same level of quality.

For the case n = 2 it holds that the $<_f$ and $<_d$ are equal, where $<_d$ is the Pareto dominance relation. So, when n = 2, the favour relation is exactly the same as the Pareto dominance relation. Relation $<_f$ can handle infeasible solutions. When an element is infeasible, the element is considered as the worst possible value. The computation time for the SC classification is $O(|P|^2 \cdot n)$, where P is the set of solutions.

5.3 Preference Order Ranking

This is a ranking procedure that exploits the definition of preference order (PO) proposed by di Pierro in [12]. The preference order definition is:

A point $\mathbf{x}^* \in \omega$ is considered efficient in order k if $f(\mathbf{x}^*)$ is not dominated by any member of P for any of the k-element subsets of the objectives. In other words, a point is efficient of order k if it is a Pareto optimal in all the $\binom{m}{k}$ subspaces of F obtained considering only k objectives at a time.

It is clear that the efficiency of order m for an MOP with exactly m objectives simply enforces Pareto optimality.

The condition of efficiency of order can be used to help reduce the number of points in a set by retaining only those that are regarded as "best compromises". In fact, it is intuitive that the less extreme components a point has, the more likely it is to be efficient of order. When the number of points selected is still considerably large, a more stringent criterion is required to sort out better solutions.

Definition (Efficiency of order k with degree z): A point $\mathbf{x}^*$ is said to be efficient of order k with degree z if it is not dominated by any member of P for exactly z out of the possible $\binom{m}{k}-$ element subsets.

At every generation t from a population P:

1.- Identify the Pareto nondominated solutions of P and group them into the subset $R^{(1)}$, which is given rank 1.
2.- Assign to the individuals of $R^{(1)}$ a rank according to a strategy based on Preference Order with Degree z and the worst given rank is w.
3.- Identify the Pareto nondominated individuals of $P \setminus R^{(1)}$ and group them into the subset $R^{(2)}$, which will be given rank $w+s$.
4.- Iterate (Step 2) and (Step 3) until $P \setminus R^{(s)} = \emptyset$, where $R^{(s)}$ is the subset that contains the worst individuals.

6 Ranking Methods with Parameters

6.1 K-optimality

Farina and Amato [15] proposed an alternative relation which takes into account the number of improved objectives between two solutions. This relation employs three quantities, $n_b(\mathbf{x}_1,\mathbf{x}_2), n_e(\mathbf{x}_1,\mathbf{x}_2)$ and $n_w(\mathbf{x}_1,\mathbf{x}_2)$, which denote the objectives where $\mathbf{x}_1$ is better, equal or worse than $\mathbf{x}_2$, respectively. Using these, the concepts of $(1-k)$-*Dominance* and k-*Optimality* are defined. A solution $\mathbf{x}_1$ $(1-k)$-dominates $\mathbf{x}_2$ if and only if

$$\begin{cases} n_e(\mathbf{x}_1,\mathbf{x}_2) < M \\ n_b(\mathbf{x}_1,\mathbf{x}_2) \geq \frac{M-n_e}{k+1} \end{cases}$$

In a similar way to Pareto optimality, a solution $\mathbf{x}^*$ is k-*optimum* if and only if there is no $\mathbf{x}$ in the decision variable space such that $\mathbf{x}$ k-dominates $\mathbf{x}^*$.

Then, this definition is fuzzificated by introducing fuzzy numbers to define $n_b(\mathbf{x}_1,\mathbf{x}_2), n_e(\mathbf{x}_1,\mathbf{x}_2)$ and $n_w(\mathbf{x}_1,\mathbf{x}_2)$. Finally, they propose a further extension that introduces a fuzzy definition for the Pareto dominance relation itself.

6.2 Contraction - Expansion

Sato, Aguirre and Tanaka [30] proposed a method to control the dominance area of solutions. This method can control the degree of expansion or contraction of the dominance area adopting a user-defined parameter S. To contract and expand the dominance area of solutions, the authors modify the fitness value for each objective function by changing the user defined parameter S_i in the following equation:

$$f_i'(\mathbf{x}) = \frac{r \cdot \sin(\omega_i + S_i \cdot \pi)}{\sin(S_i \cdot \pi)} \ \forall\ i = 1, 2, \ldots, m$$

where r is the norm of $f(\mathbf{x})$, $f_i(\mathbf{x})$ is the fitness value in the $i-th$ objective, and ω_i is the declination angle between $f(\mathbf{x})$ and $f_i(\mathbf{x})$.

If the user adopts a value of $S_i < 0.5$, the dominance area is expanded and produces a more fine grained ranking of solutions and would strengthen selection. On the other hand, if the user sets $S_i > 0.5$, the dominance area is contracted from the original one and produces a coarser ranking of solutions, weakening the selection procedure.

7 Analysis of Parameterless Ranking Methods

7.1 Ranking Distributions

One criterion to estimate the quality of a ranking method is to analyze the distribution of the ranks assigned to a set of solutions. A ranking method will favor the selection process if it is able to generate a richer range of ranks. To measure the scalability of a method with respect to the number of objectives we can determine if the shape and range of the distribution is maintained when the number of objectives is incremented. Along with the four ranking methods described in Section 5, a Pareto-based ranking method is included as a reference to compare the other ranking methods. The Pareto-based ranking method used is Fonseca and Fleming's method [16] which ranks solutions based on the Pareto dominance relation. The ranking distributions presented in this section belong to the ranking of 10000 random solutions for the problems DTLZ2 and DTLZ7 described in Table 2. For each problem we used 3, 4, 5, 8, 10, 15 and 20 objectives.

With regard to problems DTLZ2 and DTLZ7, for every number of objectives considered, the Pareto-based ranking method concentrates most of the solutions under rank 1 and the frequency for the worst ranks quickly approaches zero (see Figures 3 and 4). This behavior provides few different ranks to the selection process. In DTLZ7, for 3 objectives, about 60% of the solutions have rank 1 (see Figure 4). By observing the distributions for more objectives we can appreciate a phenomenon previously reported in the specialized literature [17, 18, 19, 36]. That is, the number of nondominated solutions (rank 1) increases quickly with the number of objectives. For example, for 8 objectives, around 80% of the solutions are nondominated in both MOPs, while for 15 and 20 objectives, all the solutions are nondominated in DTLZ7.

The distribution of the maximum ranking method (MR) for 3 and 4 objectives in DTLZ7 is similar to that of the Pareto-based ranking method. However only about 32% of the solutions have the best rank and, consequently, there is a larger range of ranks available. This behavior changes for more than 5 objectives. For that number of objectives, most of the solutions are assigned medium ranks, some solutions have the worst rank, and just a few solutions have rank 1 (only 2%). When the number of objectives is increased, the number of solutions with the best rank decreases while the number of solutions with the worst rank increases. For 10 objectives, for instance, 80% of the solutions have the worst rank and only 1% have the best rank. We believe that a distribution where the worst solutions represent the majority of the population may hinder the progress towards the Pareto front. For instance, if we carried out a tournament selection, most of the tournaments would include bad solutions. Good solutions would have small chances of survival. With respect to DTLZ2, MR produces a distribution with two tails until 10 objectives, since when the number of objectives is increased the range of ranking values is reduced (see Figure 3). For 10 or more objectives, approximately 80% of the solutions have the best rank. This means that the maximum ranking method, although scales better than the Pareto-based method, has poor scalability with respect to the other methods studied.

In both problems, the preference order ranking (POR) method also presents a skewed right distribution where the frequency of the ranks decreases slowly. Nonentheless, in contrast with the previous ranking methods, the POR method's distribution is conserved for all the objectives considered, although the range of the distribution is reduced as the number of objectives is incremented. This distribution suggests that the POR method scales well with the number of objectives.

The favour ranking method maintains well the shape and range of its rank distribution through all the objectives considered and for the two problems. However, it is the only ranking method that shows for all objectives a slightly skewed left distribution where all the ranks have a similar frequency distribution.

In DTLZ7, the AR presents a bell-shaped distribution which is more defined as the number of objectives is increased. In contrast to the other ranking methods, the range of the AR's distribution is increased with the number of objectives. With respect to DTLZ2, for a high number of objectives this method produces a slightly skewed distribution.

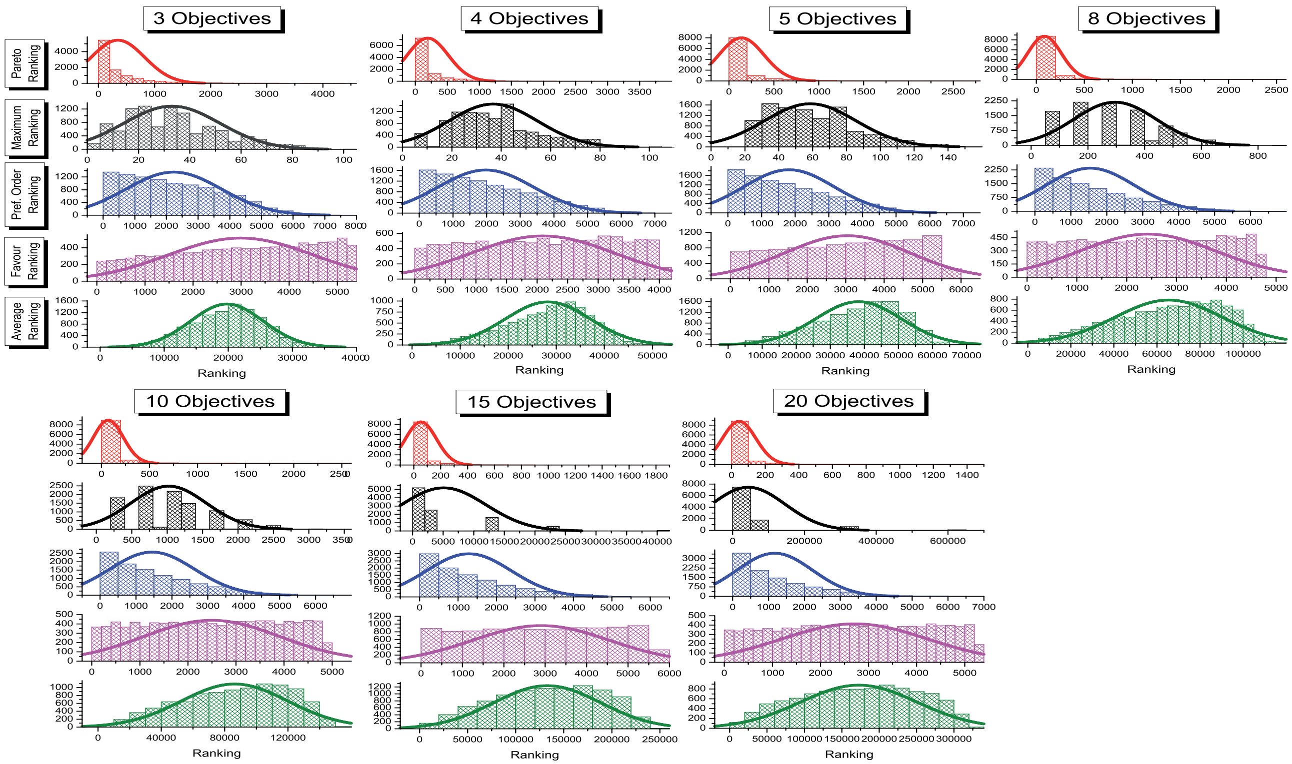

Fig. 3 A histogram that shows the density of the rankings per each different ranking method for the DTLZ2 test problem.

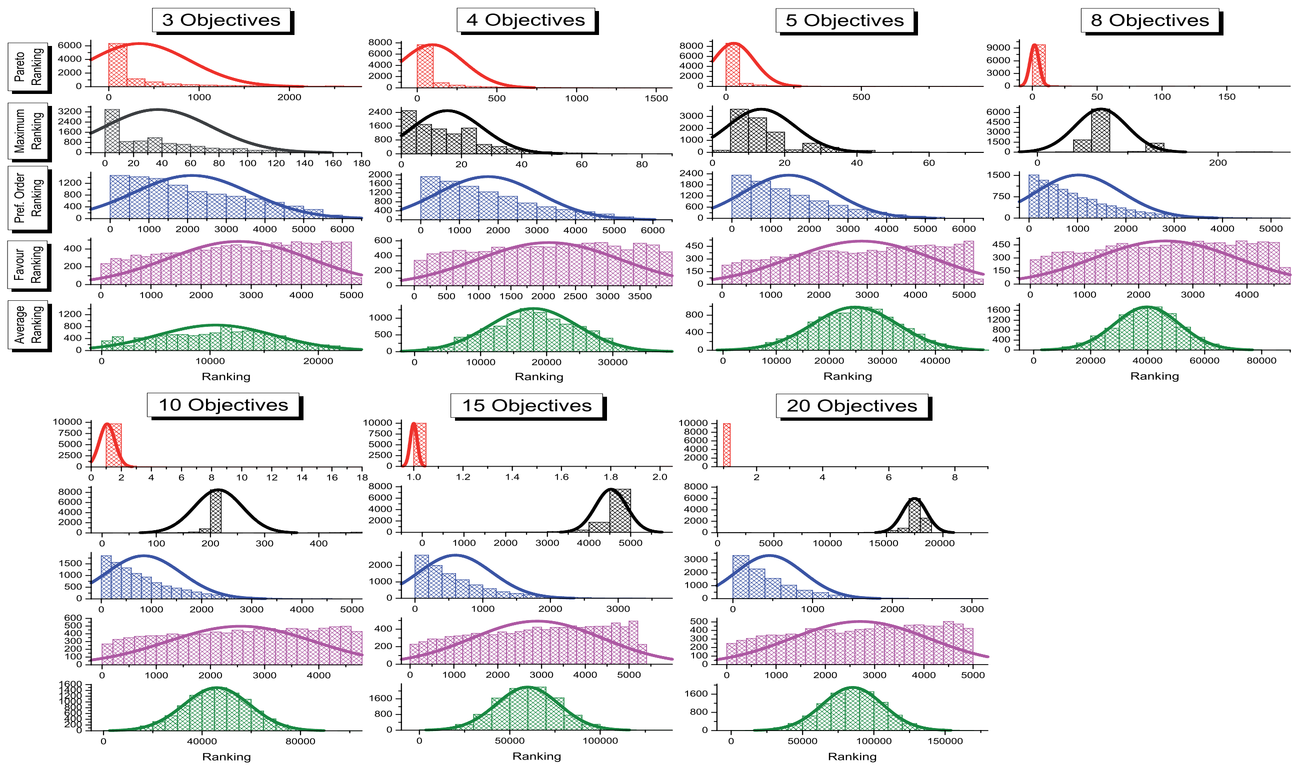

Fig. 4 A histogram that shows the density of the rankings per each different ranking method for the DTLZ7 test problem.

7.2 Ranking Landscapes

Similar to the fitness landscapes used in single-objective optimization, it is possible to visualize the behavior of a ranking method by plotting the variables against the ranking values assigned to each point of the decision space. To make this visualization possible the MOP should have at most two variables. In this study we adopted two multiobjective problems with only two variables. A 3-objective problem defined by Viennet [35] and a 5-objective problem proposed by [24]. Figures 5–9 and 10–14 show the ranking landscape generated by each ranking method in the Viennet's and Miettinen's problems respectively. Each ranking landscape is accompanied by its isocontour plot where the Pareto optimal set is shown with a shaded region.

The ranking landscape for Viennet's MOP presents a smooth and unimodal surface for all the ranking methods except for the maximum ranking method (see Figure 6). It is interesting to note that, since this method favors extreme solutions, the surface for this method has three local optima, one for each objective of the problem. Even so, the surface generated for all the methods would allow an optimizer to converge easily towards the optimal solutions.

With respect to Miettinen's MOP (see Figures 10–14), the ranking lanscape generated by all the ranking methods presents a multi-modal surface. All the surfaces have peaks and plateaus which hinder the convergence towards the optimal solutions. There are some interesting observations about these ranking landscapes. First, the isocontour plots show clearly that the ranking methods converge only to some regions of the Pareto optimal set and, consequently, some regions of the corresponding Pareto front. For example, the average and favour ranking methods only cover the upper part of the Pareto optimal set. The preference order ranking method is the only one that converges to a region more similar to the Pareto optimal set. Secondly, if we see the peaks near the optimal regions we can realize that some solutions (those in the top of the peaks) may receive worst ranks than those solutions behind the peaks but farther from the optimal region. That is, the peaks act as a barrier that keeps them from reaching the optimal solutions. It is interesting to note that the maximum ranking method generates a smoother surface with only one peak before the optimal region. This fact suggests that the maximum ranking method is useful for approaching quickly the Pareto optimal solution, although without covering the whole set.

Table 2: MOP Test Functions

Function	**Definition**
DTLZ2 [9, 10]	$F=(f_1(\mathbf{x}),f_2(\mathbf{x}),\ldots,f_M(\mathbf{x}))$, where: $$\begin{aligned} f_1(\mathbf{x}) &= (1+g(\mathbf{x_M}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2)\ldots\cos(\frac{\pi}{2}x_{M-2})\cos(\frac{\pi}{2}x_{M-1}) \\ f_2(\mathbf{x}) &= (1+g(\mathbf{x_M}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2)\ldots\cos(\frac{\pi}{2}x_{M-2})\sin(\frac{\pi}{2}x_{M-1}) \\ f_3(\mathbf{x}) &= (1+g(\mathbf{x_M}))\cos(\frac{\pi}{2}x_1)\cos(\frac{\pi}{2}x_2)\ldots\cos(\frac{\pi}{2}x_{M-2}) \\ \vdots &\quad \vdots \\ f_{M-1}(\mathbf{x}) &= (1+g(\mathbf{x_M}))\cos(\frac{\pi}{2}x_1)\sin(\frac{\pi}{2}x_2) \\ f_M(\mathbf{x}) &= (1+g(\mathbf{x_M}))\sin(\frac{\pi}{2}x_1) \\ &\text{where:} \\ g(\mathbf{x_M}) &= \sum_{x_i\in x_M}(x_i-0.5)^2 \\ x_i\in[0,1] &\ \ \forall\ \ i=1,2,\ldots,n \\ n=M+k-1 &\ \ ,\ \ k=10 \end{aligned}$$
DTLZ7 [9, 10]	$F=(f_1(\mathbf{x}),f_2(\mathbf{x}),\ldots,f_M(\mathbf{x}))$, where: $$\begin{aligned} f_1(\mathbf{x}) &= x_1 \\ f_2(\mathbf{x}) &= x_2 \\ f_3(\mathbf{x}) &= x_3 \\ \vdots &\quad \vdots \\ f_{M-1}(\mathbf{x}) &= x_{M-1} \\ f_M(\mathbf{x}) &= (1+g(\mathbf{x_M}))h(f_1,f_2,\ldots,f_{M-1},g(\mathbf{x})) \\ &\text{where:} \\ g(\mathbf{x}) &= 1+\frac{9}{\lvert\mathbf{x}_m\rvert}\sum_{x_1\in x_M}x_i \\ h(f_1,f_2,\ldots,f_{M-1},g(\mathbf{x})) &= M-\sum_{i=1}^{M-1}\left(\frac{f_i}{1+g(\mathbf{x})}(1+\sin(3\pi f_i))\right) \\ x_i\in[0,1] &\ \ \forall\ \ i=1,2,\ldots,n \\ n=M+k-1 &\ \ ,\ \ k=10 \end{aligned}$$
Viennet [35]	$F=(f_1(\mathbf{x}),f_2(\mathbf{x}),f_3(\mathbf{x}))$, where: $$\begin{aligned} f_1(x,y) &= 0.5*(x^2+y^2)+sin(x^2+y^2), \\ f_2(x,y) &= \frac{(3x-2y+4)^2}{8}+\frac{(x-y+1)^2}{27}+15, \\ f_3(x,y) &= \frac{1}{(x^2+y^2+1)}-1.1e^{(-x^2-y^2)} \\ &x\in[-3,3] \\ &y\in[-3,3] \end{aligned}$$

Table 2: (continued)

Function	Definition
Miettinen [24]	$F = (f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x}), f_4(\mathbf{x}), f_5(\mathbf{x}))$, where: $$\begin{aligned} f_1(x_1,x_2) &= f(x_1,x_2), \\ f_2(x_1,x_2) &= f(x_1-1.2, x_2-1.5), \\ f_3(x_1,x_2) &= f(x_1+0.3, x_2-3.0), \\ f_4(x_1,x_2) &= f(x_1-1.0, x_2+0.5), \\ f_5(x_1,x_2) &= f(x_1-0.5, x_2-1.7), \\ \text{where:} & \\ f(x_1,x_2) &= -u_1(x_1,x_2)-u_2(x_1,x_2)-u_3(x_1,x_2)+10 \\ u_1(x_1,x_2) &= 3(1-x_1)^2\exp(-x_1^2-(x_2+1)^2) \\ u_2(x_1,x_2) &= -10\left((1/4)x_1-x_1^3-x_2^5\right)\exp(-x_1^2-x_2^2) \\ u_3(x_1,x_2) &= (1/3)\exp\left(-(x_1+1)^2-x_2^2\right) \\ & x_1 \in [-4.9, 3.2] \\ & x_2 \in [-3.5, 6] \end{aligned}$$

8 Conclusion and Future Work

In this study we have compared some ranking methods with respect to their rank distribution and their ranking landscape which is the surface generated by the ranks assigned to solutions. The inspection of the rank distribution provided a guide to determine the scalability of the ranking methods and their possible disadvantages in the search process. The ranking landscapes allowed us to observe easily how the ranking method could assist or hinder the progress towards the Pareto optimal set.

One of our findings is that the preference order ranking is the method with the best scalability among all the methods included in this study. Also it shows a distribution similar to the one produced by Pareto-based ranking methods with two or three objectives. This behavior suggests that the introduction of preference order ranking would perform effectively if incorporated into a MOEA. Another finding is that although maximum ranking does not induce a promising ranking distribution, its ranking landscape suggests that it can be used to reach quickly some regions of the Pareto optimal set. In addition, the ranking landscapes allow us to see that each ranking method converges to a different subset of the Pareto optimal set. That is, some methods cover the Pareto front better than others. This means that if we want to find the whole Pareto front using some of these ranking methods we have to use an additional technique or to modify the method to achieve this.

As part of our future work we want to incorporate the ranking methods included in this chapter into a MOEA in order to correlate some features observed here with convergence capabilities.

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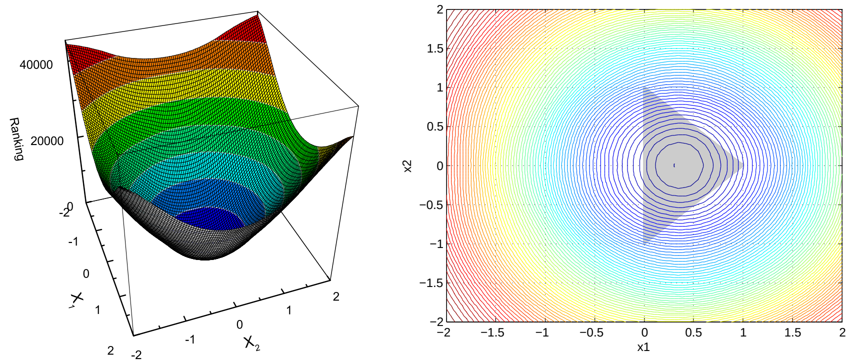

Fig. 5 Average Ranking's ranking landscape and contour - Viennet

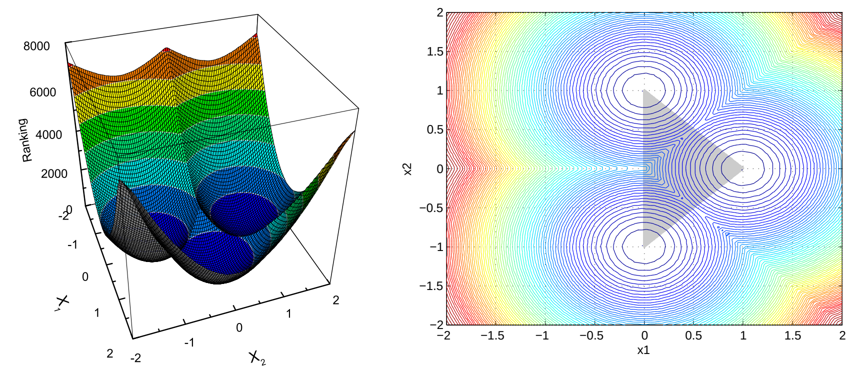

Fig. 6 Maximum Ranking - Viennet

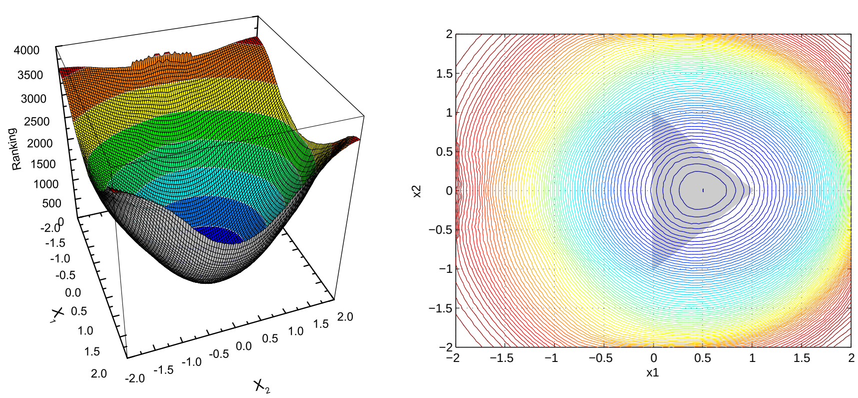

Fig. 7 Favour Ranking - Viennet

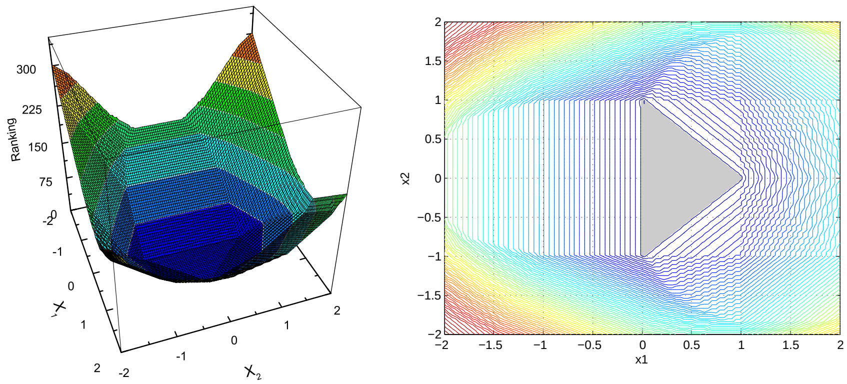

Fig. 8 Preference Order Ranking - Viennet

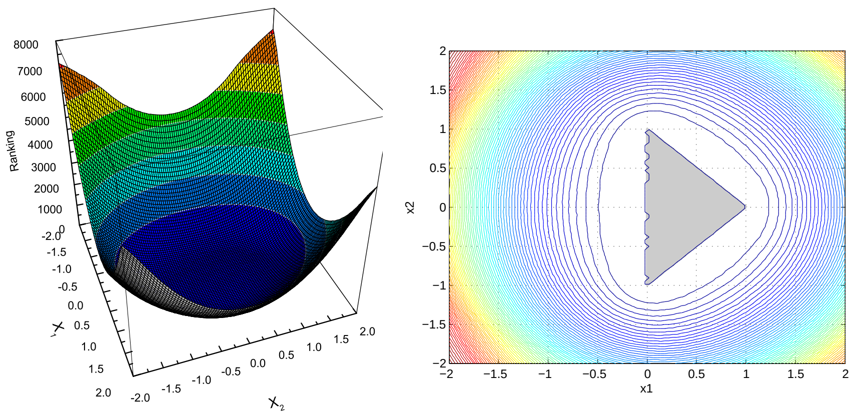

Fig. 9 Pareto Ranking - Viennet

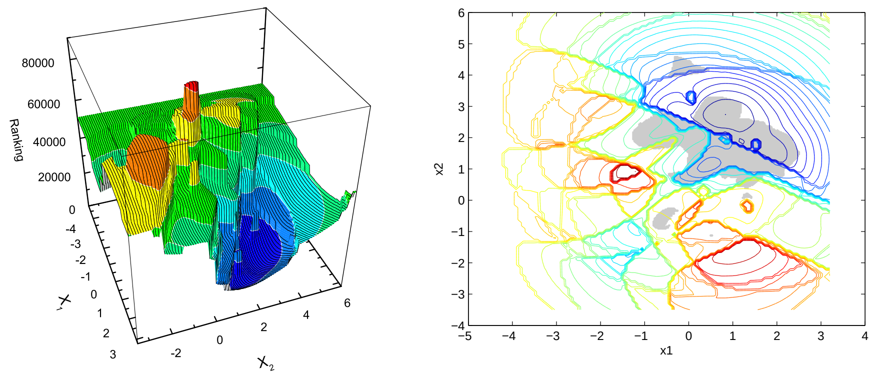

Fig. 10 Average Ranking - Miettinen

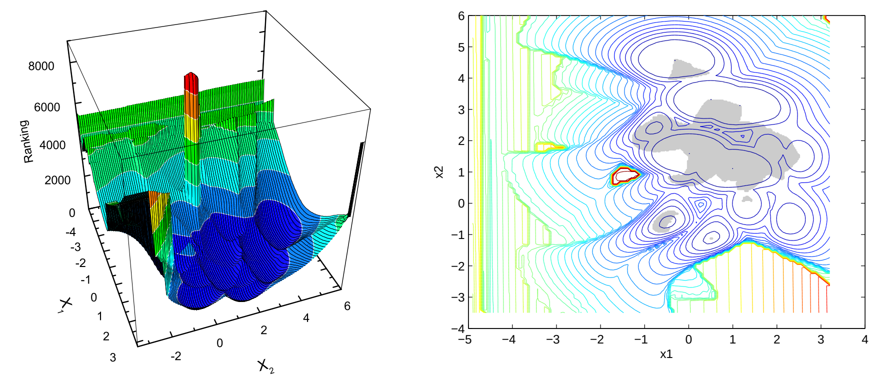

Fig. 11 Maximum Ranking - Miettinen

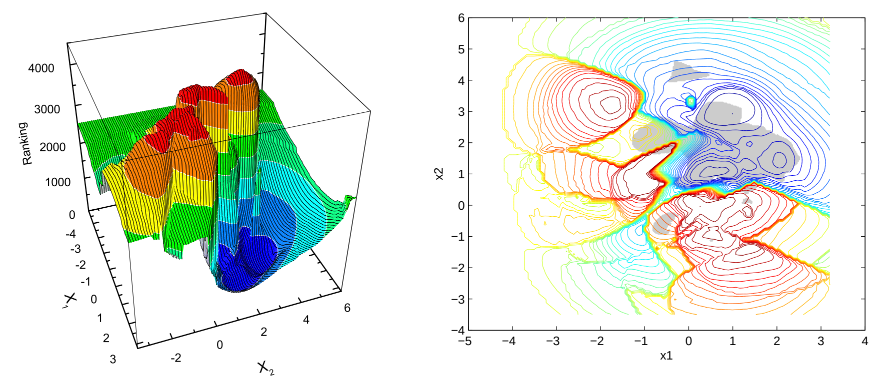

Fig. 12 Favour Ranking - Miettinen

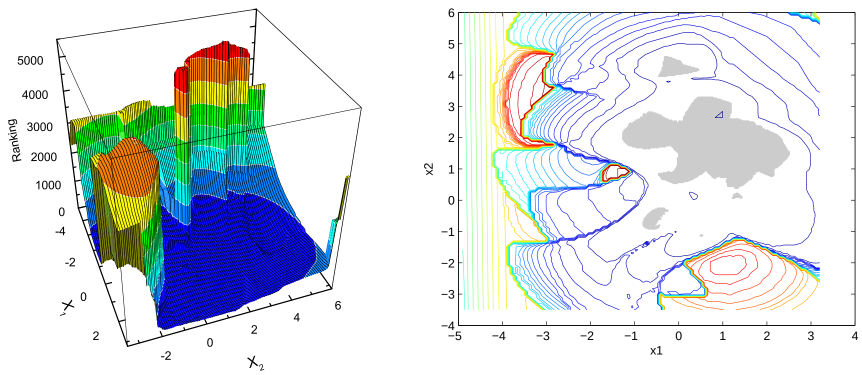

Fig. 13 Preference Order Ranking - Miettinen

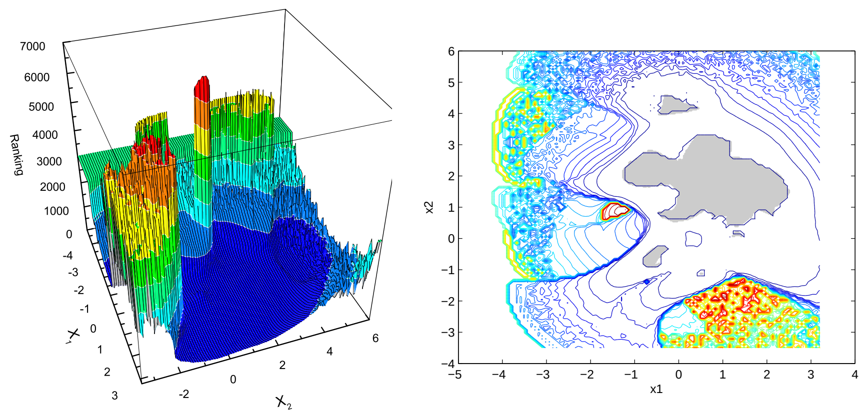

Fig. 14 Pareto Ranking - Miettinen