Alternative Fitness Assignment Methods for Many-Objective Optimization Problems

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Abstract. Pareto dominance (PD) has been the most commonly adopted relation to compare solutions in the multiobjective optimization context. Multiobjective evolutionary algorithms (MOEAs) based on PD have been successfully used in order to optimize bi-objective and three-objective problems. However, it has been shown that Pareto dominance loses its effectiveness as the number of objectives increases and thus, the convergence behavior of approaches based on this concept decreases. This paper tackles the MOEAs' scalability problem that arises as we increase the number of objective functions. In our study, we perform a comparative study of some of the state-of-the-art fitness assignment methods available for multiobjective optimization in order to analyze their ability to guide the search process in high-dimensional objective spaces.

1 Introduction

Evolutionary algorithms (EAs) draw inspiration from the process of natural evolution in order to evolve progressively a population of individuals (*i.e.*, potential solutions to the optimization problem) through the application of a series of probabilistic processes. As a population-based approach, EAs are suitable alternatives to solve problems with two or more objectives (the so-called *multiobjective optimization problems*, or MOPs for short), since they are able to explore simultaneously different regions of the search space and to obtain several points from the trade-off surface in a single run. Since the mid-1980s, the field of evolutionary multiobjective optimization (EMO, for short) has grown and a wide variety of multiobjective EAs (or MOEAs) have been proposed so far.

Despite the considerable volume of research on EMO, most of these efforts have been focused on bi-objective or three-objective problems. Recently, the EMO community started to explore the scalability of MOEAs with respect to the number of objective functions. As a result, several studies have shown that even the most popular MOEAs fail to converge to the trade-off surface in high-dimensional objective spaces [16, 12, 10]. MOPs having more than 3 objectives are referred to as *many-objective optimization problems* in the specialized literature [7].

EAs requires a function which measures the fitness of solutions in order to identify the best candidates to guide the search process. When dealing with a single-objective problem, such fitness function is usually related to the function to be optimized. However, when solving MOPs it is required an additional mechanism to map the multi-objective space into a single dimension in order to allow a direct comparison among solutions; this mechanism is known as the *fitness assignment process*[3] [11].

Pareto dominance (PD) has been the most commonly adopted relation to discriminate among solutions in the multiobjective context, and it has been the basis to develop most of the MOEAs proposed so far. However, PD loses its discrimination potential with the increase in the number of objectives and thus, decreases the convergence ability of approaches based on this concept. With the aim of clarifying this point, Figure 1 shows how the number of nondominated solutions (*i.e.*, equally good solutions for PD) grows in a population in relation with the number of objectives as the search progresses. This experiment was performed using two well-known scalable test problems, namely DTLZ1 and DTLZ6 [4]. The data in Figure 1 corresponds to the mean of 31 independent runs of a generic MOEA (described in Section 3) with a population of 100 individuals.

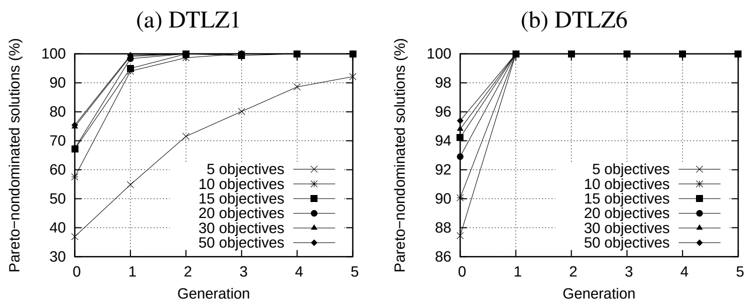

Fig. 1: Proportion of Pareto-nondominated solutions with respect to the number of objectives.

From Figure 1, we can clearly see that an increment of the number of objectives raises the proportion of nondominated individuals even in the case of the initial population (generation 0) which is randomly generated. This problem becomes more evident as the search progresses and the population is rapidly saturated with nondominated solutions. When the whole population becomes nondominated it is not possible to discriminate among solutions and thus, the search process weakens since it is performed practically at random.

It should be clear how important is to devise alternative approaches to rank solutions when dealing with many-objective problems. This paper tackles the MOEAs' scalability problem that arises when the number of objectives is increased, by performing a comparative study of some state-of-the-art alternative approaches to PD. In our study, we incorporate the considered approaches into a generic MOEA in order to investigate their convergence ability and scalability with respect to the number of objectives.

The remainder of this document is structured as follows: Section 2 describes the studied approaches. In Section 3, we present the results of the performed comparative study. Finally, Section 4 provides our conclusions as well as some possible directions for future research.

[3] In this study, we will use indistinctly the terms fitness and rank to refer to the value which expresses the quality of solutions and allows to compare them with respect to each other.

2 Multiobjective fitness assignment methods

In this study, we assume that all objectives are equally important and, without loss of generality, we will refer only to minimization problems. Here, we are interested in solving *many-objective problems* with the following form:

$$\begin{aligned} &\text{Minimize} \;\; \mathbf{F}(\mathbf{X}_i) = [f_1(\mathbf{X}_i), f_2(\mathbf{X}_i), \ldots, f_M(\mathbf{X}_i)]^T \\ &\text{subject to} \;\; \mathbf{X}_i \in \mathcal{F} \end{aligned} \tag{1}$$

where $\mathbf{X}_i$ is a *decision vector* (containing *decision variables*), $\mathbf{F}(\mathbf{X}_i)$ is the M-dimensional *objective vector* ($M > 3$), $f_m(\mathbf{X}_i)$ is the m-th objective function, and $\mathcal{F}$ is the feasible region delimited by the problem's constraints.

Also, from now on, we will use the ranking procedure proposed by Fonseca and Fleming [9] for all the approaches described herein, except for those for which a different ranking method is explicitly given. Fonseca and Fleming proposed to rank a solution $\mathbf{X}_i$ as follows:

$$\text{rank}(\mathbf{X}_i) = 1 + |\{\mathbf{X}_j : \mathbf{X}_j \prec \mathbf{X}_i\}| \tag{2}$$

where $\mathbf{X}_j \prec \mathbf{X}_i$ denotes that solution $\mathbf{X}_j$ dominates (is better than) $\mathbf{X}_i$ according to a preference relation $\prec$. $\prec$ was originally proposed for Pareto dominance.

2.1 Pareto dominance

Pareto dominance (PD) was proposed by Vilfredo Pareto [15] and is defined as follows: given two solutions $\mathbf{X}_i, \mathbf{X}_j \in \mathcal{F}$, we say that $\mathbf{X}_i$ Pareto-dominates $\mathbf{X}_j$ ($\mathbf{X}_i \prec_P \mathbf{X}_j$) if and only if:

$$\begin{aligned} &\forall m \in \{1,2,\ldots,M\} : f_m(\mathbf{X}_i) \leq f_m(\mathbf{X}_j) \;\; \wedge \\ &\exists m \in \{1,2,\ldots,M\} : f_m(\mathbf{X}_i) < f_m(\mathbf{X}_j) \end{aligned} \tag{3}$$

2.2 Ranking composition methods

Ranking composition methods (RCM) extract the separated fitnesses of every solution into a list of fitness values for each objective. These lists are then individually sorted, resulting in a set of different ranking positions for every solution for each objective. The ranking positions of a solution $\mathbf{X}_i$ are given by the vector $\mathbf{R}(\mathbf{X}_i) = [r_1(\mathbf{X}_i), r_2(\mathbf{X}_i), \ldots, r_M(\mathbf{X}_i)]^T$, where $r_m(\mathbf{X}_i)$ is the rank of $\mathbf{X}_i$ for the m-th objective. Finally, the different ranking positions of an individual are composed into a single ranking which reflects the candidate solutions' quality [1]. Assuming a previous calculation of $\mathbf{R}(\mathbf{X}_i)$ for each solution $\mathbf{X}_i$, we describe below some RCM reported in the specialized literature.

Average Ranking (AR). This method was proposed by Bentley and Wakefield [1]. The global rank of an individual $\mathbf{X}_i$ is given by:

$$\text{rank}(\mathbf{X}_i) = \sum_{m=1}^{M} r_m(\mathbf{X}_i) \tag{4}$$

Maximum Ranking (MR). Bentley and Wakefield [1] also proposed a method in which the global rank of an individual corresponds to its best ranking position:

$$\text{rank}(\mathbf{X}_i) = \min_{m=1}^{M} r_m(\mathbf{X}_i) \tag{5}$$

2.3 Relaxed forms of dominance

Some authors have developed some alternatives to PD in order to allow a finer grain discrimination among solutions. Relaxed forms of dominance (RFD), make possible for a solution $\mathbf{X}_i$ to dominate another solution $\mathbf{X}_j$ even in cases when $\mathbf{X}_i$ is Pareto-dominated by $\mathbf{X}_j$. Generally, RFD can accept a detriment in some objectives if the solution presents a considerable improvement in the other objectives. Next, we describe some RFD reported in the literature.

Approaches that require parameters

The approaches described below require the fine-tuning of at least one parameter which sometimes involves in-depth knowledge or understanding of the method and the problem to be solved. This specification can be seen as a drawback, since it can reduce the application range of such approaches.

α-domination Strategy (AD). Ikeda *et al.* [13] proposed a RFD to deal with what they called *dominance resistant solutions* (DRS), *i.e.*, solutions that are extremely inferior to others in at least one objective, but hardly-dominated. The idea of AD is setting upper/lower bounds of trade-off rates between two objectives, in order to allow $\mathbf{X}_i$ to dominate $\mathbf{X}_j$ if $\mathbf{X}_i$ is slightly inferior in an objective but largely superior in some other objectives.

A solution $\mathbf{X}_i$ α-dominates solution $\mathbf{X}_j$ ($\mathbf{X}_i \prec_\alpha \mathbf{X}_j$) if and only if:

$$\begin{aligned} &\forall m \in \{1,2,\ldots,M\} : g_m(\mathbf{X}_i,\mathbf{X}_j) \leq 0 \quad \wedge \\ &\exists m \in \{1,2,\ldots,M\} : g_m(\mathbf{X}_i,\mathbf{X}_j) < 0 \end{aligned} \tag{6}$$

where

$$g_m(\mathbf{X}_i,\mathbf{X}_j) = f_m(\mathbf{X}_i) - f_m(\mathbf{X}_j) + \sum_{n \neq m} \alpha_{mn}(f_n(\mathbf{X}_i) - f_n(\mathbf{X}_j)) \tag{7}$$

and α_{mn} is the trade-off rate between objectives m and n. If $\alpha_{mn} = 0$ for all pairs of objectives, AD enforces PD. In [13], parameters α_{mn} were set to a constant $c = \{\frac{1}{3}, \frac{1}{9}, \frac{1}{100}\}$ in all $m \neq n$. Since $\frac{1}{3}$ was the value which allowed the best performance in [13], this value will be used for this study.

k-dominance (KD). Farina and Amato [8] proposed a dominance relation which takes into account the number of objectives where a solution $\mathbf{X}_i$ is better, equal and worse than another solution $\mathbf{X}_j$. For this purpose, the authors defined respectively the following functions:

$$n_b(\mathbf{X}_i,\mathbf{X}_j) = |\{m : f_m(\mathbf{X}_i) < f_m(\mathbf{X}_j)\}| \quad m = 1,2,\ldots,M \tag{8}$$
$$n_e(\mathbf{X}_i,\mathbf{X}_j) = |\{m : f_m(\mathbf{X}_i) = f_m(\mathbf{X}_j)\}| \quad m = 1,2,\ldots,M \tag{9}$$
$$n_w(\mathbf{X}_i,\mathbf{X}_j) = |\{m : f_m(\mathbf{X}_i) > f_m(\mathbf{X}_j)\}| \quad m = 1,2,\ldots,M \tag{10}$$

For simplicity, we will refer to functions in Equations (8), (9) and (10) as n_b, n_e and n_w, respectively. Given two solutions $\mathbf{X}_i$ and $\mathbf{X}_j$, we say that $\mathbf{X}_i$ *k*-dominates[4] $\mathbf{X}_j$ ($\mathbf{X}_i \prec_k \mathbf{X}_j$) if and only if:

$$n_e < M \wedge n_b \geq \frac{M - n_e}{k+1} \tag{11}$$

where $0 \leq k \leq 1$. If k=0, KD and PD relations' discrimination would be equivalent. The strictness of KD depends on the value chosen for k. In this study the value $k = 1$ will be used in order to enhance discrimination among solutions.

Volume dominance (VD). This dominance relation was proposed by Le and Landa-Silva [14]. VD is based on the volume of the objective space that a solution dominates. The dominated volume of $\mathbf{X}_i$ is defined as the region R for which all its feasible solutions are dominated by $\mathbf{X}_i$. We need to define a reference point r such that it is dominated by all solutions in R. The dominated volume of a solution $\mathbf{X}_i$ with respect to the reference point $r = [r_1, r_2, \ldots, r_M]^T$ is given by:

$$V(\mathbf{X}_i, r) = \prod_{m=1}^{M} (r_m - f_m(\mathbf{X}_i)) \tag{12}$$

To establish the dominance relationship of two solutions $\mathbf{X}_i$ and $\mathbf{X}_j$, we need to compare their dominated volumes to the *shared dominated volume* (SV), *i.e.*, the volume dominated by both solutions. The SV is defined as follows:

$$SV(\mathbf{X}_i,\mathbf{X}_j, r) = \prod_{i=1}^{M} (r_i - max(f_m(\mathbf{X}_i), f_m(\mathbf{X}_j))) \tag{13}$$

It is said that $\mathbf{X}_i$ volume-dominates $\mathbf{X}_j$ ($\mathbf{X}_i \prec_V \mathbf{X}_j$) for a ratio rSV if either (14) or (15) holds.

$$V(\mathbf{X}_j, r) = SV(\mathbf{X}_i,\mathbf{X}_j, r) \wedge V(\mathbf{X}_i, r) > SV(\mathbf{X}_i,\mathbf{X}_j, r) \tag{14}$$

$$V(\mathbf{X}_i, r) > V(\mathbf{X}_j, r) > SV(\mathbf{X}_i,\mathbf{X}_j, r) \wedge \frac{V(\mathbf{X}_i, r) - V(\mathbf{X}_j, r)}{SV(\mathbf{X}_i,\mathbf{X}_j, r)} > rSV \tag{15}$$

A small rSV indicates that a small difference between the dominated volumes of two solutions is enough to establish preferences between them. The authors suggested

[4] In [8] the term $(1-k)$-dominates is used, but for simplicity we will use k-dominates instead.

us that a value in the range [0.05, 0.15] is reasonable for rSV; we will use $rSV = 0.1$. In our experimental study we will apply this method to the normalized objectives within the range $[0,1]$ (see details in Section 3) and thus, the reference point could be any point r such that $r_m > 1$. We'll use $r = 1.1^M$ for this study.

Contraction/Expansion of Dominance Area (CE). Sato *et al.* [17] proposed a method to strengthen or weaken the selection process by expanding or contracting the solutions' dominance area. The fitness value of a solution $\mathbf{X}_i$ for each objective function is modified as follows:

$$f'_m(\mathbf{X}_i) = \frac{r \cdot \sin(\omega_m + S_m \cdot \pi)}{\sin(S_m \cdot \pi)} \quad \forall m \in \{1, 2, \ldots, M\} \tag{16}$$

where r is the norm of vector $\mathbf{F}(\mathbf{X}_i)$ and ω_m is the declination angle between $\mathbf{F}(\mathbf{X}_i)$ and $f_m(\mathbf{X}_i)$, which can be calculated as $\omega_m = f_m(\mathbf{X}_i)/r$. S_m is a user defined parameter which allows to control the dominance area of $\mathbf{X}_i$ for the m-th dimension. The possible values for S_m lie in the range $[0.25, 0.75]$. If $S_m = 0.5$, then $f'_m(\mathbf{X}_i) = f_m(\mathbf{X}_i)$. Otherwise, if $S_m > 0.5$ the dominated area is contracted, producing a coarser ranking of solutions and would weaken the selection process. On the other hand, $S_m < 0.5$ expands the dominance area and would strengthen the selection by producing a more fine grained ranking of solutions. It is clear that for many-objective problems, we are interested in expanding the dominance area of solutions in order to achieve a richer ordering of preferences among them. For this study we adopted the value $S_m = 0.25$ for all $m \in \{1, 2, \ldots, M\}$.

Parameter-less approaches

Unlike the above methods, the operation of the approaches described in this section does not depend on any parameters' fine-tuning, which expands their applicability and facilitates their understanding and implementation.

***L*-dominance (LD).** This dominance relation was proposed by Zou *et al.* [19]. Similar to the KD relation, LD considers functions n_b (8), n_e (9) and n_w (10), which count the number of objectives in which a solution $\mathbf{X}_i$ is respectively better, equal and worse than another solution $\mathbf{X}_j$. According to LD, we can say that $\mathbf{X}_i$ L-dominates $\mathbf{X}_j$ ($\mathbf{X}_i \prec_L \mathbf{X}_j$) if and only if:

$$n_b - n_w = L > 0 \quad \wedge \quad \|\mathrm{f}(\mathbf{X}_i)\|_p < \|\mathrm{f}(\mathbf{X}_j)\|_p \text{ (for certain } p) \tag{17}$$

where $\|\mathbf{F}(\mathbf{X}_i)\|_p$ is the p-norm of a solution $\mathbf{X}_i$. The value $p = 1$ is used in this study.

Favour Relation (FD). In this alternative dominance relation, proposed by Drechsler *et al.* [6], a solution $\mathbf{X}_i$ is said to dominate another solution $\mathbf{X}_j$ ($\mathbf{X}_i \prec_f \mathbf{X}_j$) if and only if:

$$|\{m : f_m(\mathbf{X}_i) < f_m(\mathbf{X}_j)\}| > |\{n : f_n(\mathbf{X}_j) < f_n(\mathbf{X}_i)\}| \quad \text{for } m, n \in \{1, 2, \ldots, M\} \tag{18}$$

Since FD is not a transitive relation (consider solutions $\mathbf{X}_i$=(8,7,1), $\mathbf{X}_j$=(1,9,6) and $\mathbf{X}_k$=(7,0,9); it is clear that $\mathbf{X}_i \prec_f \mathbf{X}_j \prec_f \mathbf{X}_k \prec_f \mathbf{X}_i$), authors proposed to rank solutions as follows: to use a graph representation for the relation, where each solution is a node and the preferences are given by edges, in order to identify the *Strongly Connected Components* (SCC). A SCC groups all elements which are not comparable to each other (as the cycle of solutions in the above example). A new cycle-free graph is constructed using the obtained SCCs, such that it would be possible to establish an order by assigning the same rank to all solutions that belong to the same SCC.

Preference order ranking (PO). di Pierro *et al.* proposed an strategy that ranks a population according to the order of efficiency of solutions [5]. An individual $\mathbf{X}_i$ is considered efficient of order k if it is not Pareto-dominated by any other individual for any of the $\binom{M}{k}$ subspaces where are considered only k objectives at a time. Efficiency of order M for a MOP with exactly M objectives simply corresponds to the original Pareto optimality definition.

If $\mathbf{X}_i$ is efficient of order k, then it is efficient of order $k+1$. Analogously, if $\mathbf{X}_i$ is not efficient of order k, then it is not efficient of order $k-1$. Given these properties, the order of efficiency of a solution $\mathbf{X}_i$ is the minimum k value for which $\mathbf{X}_i$ is efficient. Formally:

$$order(\mathbf{X}_i) = \min_{k=1}^{M}(k : isEfficient(\mathbf{X}_i, k)) \qquad (19)$$

where $isEfficient(\mathbf{X}_i, k)$ is to be true if $\mathbf{X}_i$ is efficient of order k. The order or efficiency can be used to rank solutions. The smaller the order of efficiency an individual has, the better this is.

In [5] it is proposed to use this strategy in combination with a PD-based ranking procedure, in such a way that the order of efficiency can be used to discriminate among solutions classified with the same rank according to PD. However, since it is known that for many-objective problems the whole population rapidly becomes nondominated (all solutions share the same rank), in this study we rank solutions by using the order of efficiency alone.

3 Experimental results

The different ranking methods described in Section 2 were incorporated into a generic MOEA in order to investigate their convergence ability as the number of objectives increases. Figure 2 describes the implemented MOEA's workflow. Initially, a parent population of N individuals is randomly generated. Then, this population is ranked and selection is performed in order to identify those individuals which are to be reproduced. A children population of N new individuals is generated by applying variator operators over the selected individuals. Finally, parent and children populations are combined and ranked in order to select the N best individuals to survive and form the new parent population (elitist MOEA [2]). The ranking step (which is remarked in Figure 2) is where the different studied approaches were incorporated.

The implemented operators are: *binary tournament selection* based on the rank of solutions. *Simulated binary crossover* ($\eta_c = 15$) with probability of 1. *Polynomial mutation* ($\eta_m = 20$) with probability of $1/n$, where n is the number of decision variables.

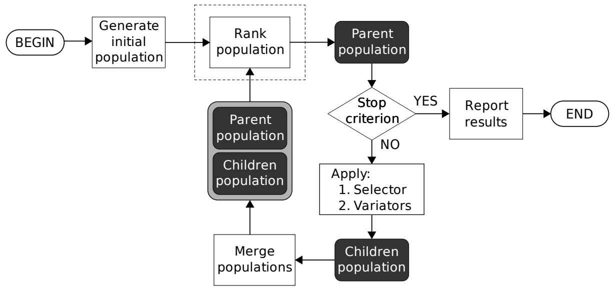

Fig. 2: Generic MOEA's workflow.

We used a population of $N = 100$ individuals and 300 generations for all experiments. In order to avoid alterations in the behavior of the studied methods, we did not use any additional mechanism to maintain diversity in the population.

The different studied approaches were applied to the normalized objective values: $f'_m(\mathbf{X}_i) = \frac{f_m(\mathbf{X}_i) - GMIN_m}{GMAX_m - GMIN_m}$ for $m = 1, 2, ..., M$, where $GMAX_m$ and $GMIN_m$ are the maximum and minimum known values for the m-th objective. Since we know *a priori* that for the adopted set of test problems $GMIN_m = 0$, we simply normalized the objectives as follows: $f'_m(\mathbf{X}_i) = \frac{f_m(\mathbf{X}_i)}{GMAX_m}$ for $m = 1, 2, ..., M$.

Problems DTLZ1 and DTLZ6 [4] were selected for our experimental study. These test functions can be scaled to any number of objectives and decision variables. The total number of variables in these problems is $n = M + k - 1$, where M is the number of objectives. k is a difficulty parameter and was set to $k = 5$ for DTLZ1 and $k = 10$ for DTLZ6. In this study, we consider instances with $M = \{5, 10, 15, 20, 30, 50\}$ objectives. However, since PO becomes computationally expensive as the number of objectives increases, we only applied it for instances with up to 20 objectives.

As a convergence measure, the average distance of Pareto-nondominated solutions in the approximation set obtained by the MOEA from the Pareto front was computed [3]. Since equations defining the Pareto front are known for the test problems adopted, the convergence measure was analytically determined (without using any reference set) [12, 18].

Tables 1 and 2 show the obtained results when the different methods were applied to problems DTLZ1 and DTLZ6, respectively. These tables show the average and standard deviation of the convergence measure for 31 independent trails of each experiment.

Table 1: Average and standard deviation of the achieved convergence in 31 runs for DTLZ1.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
PD	1.363 ± 1.072	17.94 ± 14.67	6.740 ± 7.553	6.615 ± 7.254	5.687 ± 5.448	2.845 ± 2.301
AR	**0.000 ± 0.000**	**0.000 ± 0.000**	0.005 ± 0.023	0.019 ± 0.041	0.110 ± 0.117	0.378 ± 0.281
MR	45.40 ± 22.77	32.37 ± 12.13	26.62 ± 10.22	29.89 ± 12.32	18.92 ± 10.37	15.83 ± 8.824
AD	0.002 ± 0.002	0.001 ± 0.002	0.007 ± 0.024	0.005 ± 0.020	0.013 ± 0.031	0.091 ± 0.074
KD	**0.000 ± 0.000**	0.025 ± 0.134	0.022 ± 0.047	0.033 ± 0.058	0.128 ± 0.107	0.480 ± 0.329
VD	0.582 ± 0.355	0.464 ± 0.468	0.515 ± 0.384	0.400 ± 0.415	0.388 ± 0.374	0.323 ± 0.234
CE	0.002 ± 0.001	0.002 ± 0.001	**0.002 ± 0.002**	**0.002 ± 0.001**	0.008 ± 0.022	0.032 ± 0.059
LD	**0.000 ± 0.000**	**0.000 ± 0.000**	0.004 ± 0.023	0.004 ± 0.020	**0.006 ± 0.023**	**0.030 ± 0.035**
FD	0.001 ± 0.001	0.047 ± 0.151	0.121 ± 0.237	0.277 ± 0.492	0.829 ± 1.131	2.859 ± 3.330
PO	0.750 ± 0.691	1.879 ± 4.646	1.121 ± 1.302	0.853 ± 0.816	-	-

From Table 1 we can highlight that, for DTLZ1, LD, CE and AD showed the best convergence ability as the number of objectives increases, whereas MR obtained the worst average convergence for all instances of this problem.

Table 2: Average and standard deviation of the achieved convergence in 31 runs for DTLZ6.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
PD	6.525 ± 0.402	8.485 ± 0.350	8.584 ± 0.353	8.646 ± 0.352	8.783 ± 0.467	8.870 ± 0.285
AR	0.150 ± 0.044	10.00 ± 0.000	10.00 ± 0.000	10.00 ± 0.000	10.00 ± 0.000	10.00 ± 0.000
MR	8.607 ± 0.349	9.543 ± 0.233	9.820 ± 0.155	9.828 ± 0.096	9.887 ± 0.057	9.874 ± 0.050
AD	0.074 ± 0.028	0.553 ± 0.069	0.677 ± 0.109	0.659 ± 0.091	0.706 ± 0.093	0.679 ± 0.081
KD	**0.063 ± 0.032**	10.00 ± 0.000	10.00 ± 0.000	10.00 ± 0.000	10.00 ± 0.000	10.00 ± 0.000
VD	0.106 ± 0.035	0.102 ± 0.038	0.152 ± 0.089	0.213 ± 0.140	0.447 ± 0.313	0.634 ± 0.371
CE	0.081 ± 0.029	**0.089 ± 0.030**	**0.080 ± 0.027**	**0.089 ± 0.030**	**0.094 ± 0.032**	**0.091 ± 0.028**
LD	0.079 ± 0.029	7.416 ± 0.237	7.149 ± 0.319	7.105 ± 0.338	6.744 ± 0.326	6.218 ± 0.354
FD	5.377 ± 2.361	9.902 ± 0.135	9.515 ± 0.166	9.864 ± 0.096	9.742 ± 0.106	9.641 ± 0.112
PO	5.836 ± 0.644	9.847 ± 0.106	9.968 ± 0.041	9.994 ± 0.007	-	-

Problem DTLZ6 (Table 2) imposes higher convergence difficulties for most of the studied approaches. On the one hand, CE was the only method which achieved relatively low values for the convergence measure in all the instances of this problem. On the other hand, AR and KD seem to be the most affected methods by the DTLZ6's difficulties, since these obtained the worst performance in almost all instances.

Results confirm that Pareto dominance is not able to effectively guide the search in many-objective scenarios. However, the achieved convergence of MR is even worse than that of PR in all cases. In our opinion, this is because MR tends to favor extreme solutions, *i.e.*, it prefers solutions with the best performance for some objectives but without taking into account their assessment in the rest of the objectives. With the aim of clarifying this point, we present the following example. If in a 20-objective MOP, $\mathbf{X}_i$ is the solution with the best performance with respect to the first objective, but it is the worst solution for the remainder 19 objectives, $\mathbf{X}_i$ would be classified with the best rank by MR. We consider MR as the worst of the studied alternatives.

In general, according to our experimental observations, MR, PD, PO and FR are the four methods with the worst performance. On the other hand, results suggest that CE provides the best convergence properties and the most stable behavior as the number of objectives increases.

Additionally, we investigated the convergence speed achieved by the MOEA when using the different ranking schemes of our interest. Figures 3 and 4 show for DTLZ1 and DTLZ6, respectively, the average convergence of 31 runs as the search progresses. Due to space limitations, we only show results for the 20-objective instances, since 20 is the maximum number of objectives for which we performed experiments for all the studied methods. The data shown in Figures 3 and 4 was plotted in logarithmic scale, in order to highlight the differences in the results obtained using each method.

Figure 3 shows that CE, LD and AD performed the best for problem DTLZ1. We can clearly see that these three methods had an accelerated convergence, since during the first 50 generations they reached relatively low values for the convergence measure.

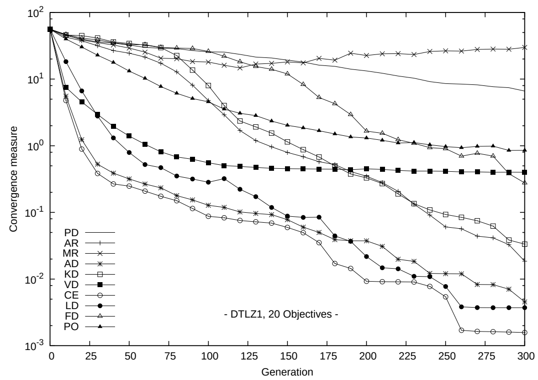

Fig. 3: Convergence at different search stages for DTLZ1 problem with 20 objectives.

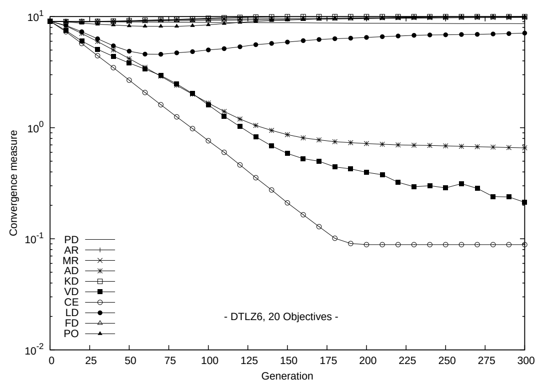

Fig. 4: Convergence at different search stages for DTLZ6 problem with 20 objectives.

Regarding the DTLZ6 problem, Figure 4 shows that, as stated before, most of the studied alternatives did not achieve good convergence results. CE, VD and AD (in this order) showed the best convergence properties and a high convergence speed during the first 130 generations.

4 Conclusions and Future Work

Since the performance of Pareto-based MOEAs deteriorates as the number of objectives increases, it is necessary to identify alternative approaches to establish preferences among solutions in many-objective scenarios. In this paper, we performed a comparative study of some state-of-the-art approaches of this sort in order to investigate their ability to guide the search process in high dimensional objective spaces.

Due to space limitations we only considered two test cases. However, it is of our interest to extend these experiments to a larger set of test functions as well as to adopt real-world many-objective problems in order to generalize our results.

Since the performance of some of the studied approaches depends of a proper parameters' fine-tuning, as part of our future work we want to investigate the influence of the parameter's settings on the behavior of such approaches.

In this paper we focused on the convergence properties of different ranking schemes. However, an important issue of MOEAs is to converge to a set of well-spread solutions. Therefore, we also want to extend our experiments in order to study the distribution of the approximation set achieved by each of the studied methods.

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