Preference Incorporation to Solve Many-Objective Airfoil Design Problems

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***Abstract*—In this paper, we assess the convenience of applying a previously proposed interactive method to solve three aerodynamic airfoil shape optimization problems with 2, 3, and 6 objectives, respectively. The expensive simulations required to evaluate the objective functions makes these problems an excellent example in which the use of interactive methods is very advantageous. First, the search can be focused on the decision maker's region of interest, saving this way, valuable function evaluations. Second, the preference relation used in the interactive method helps to deal with a large number of objectives since it is able to rank incomparable nondominated solutions. The experimental evaluation reveals that in the three problems studied, the interactive method achieved a better final solution than a traditional *a posteriori* method with no preferences. Nevertheless, in the problem with 6 objectives, only 3 of them were improved. A possible explanation for this is that local optima become harder to overcome when the size of the region of interest is very small. Additional experiments confirmed that the convergence is deteriorated if very small regions of interest are used.**

***Index Terms*—Preference incorporation, many-objective optimization, airfoil shape optimization problems.**

I. INTRODUCTION

Solving a Multiobjective Optimization Problem (MOP) involves two main tasks: search and decision making (preference articulation). Unlike single-objective optimization, in multiobjective optimization, instead of a single optimum, there is usually a set of optimal solutions that represent different degrees of trade-off among the objectives. This set of solutions is known as the *Pareto optimal set*. Thus, a possible procedure to solve a MOP is to obtain an approximation of the Pareto optimal set, and then let a decision maker (DM) to select the most preferred solution from the approximation set. This is, however, not the only way to incorporate user's preferences. There are several other methodologies for incorporating preferences from the DM into the search process. Depending on the moment at which the DM is required to provide preference information, three approaches to solve a MOP [1] are defined:

1) **A priori approaches**. The preference information is incorporated before the search, and then, the search is concentrated on the region of interest defined by the preferences.
2) **Interactive approaches**. The DM is iteratively asked to proportionate his preferences during the search in order to progressively refine the region of interest.
3) **A posteriori approaches**. The preference information is incorporated after a representative sample of the Pareto set has been obtained.

Traditionally, most Multiobjective Evolutionary Algorithms (MOEAs) have been designed as *a posteriori approaches*. In other words, their task is to obtain an approximation of the entire Pareto front (see e.g., [2], [3], [4]).

Besides providing a sample of the Pareto front to select a final solution, knowing the Pareto front might be insightful for other reasons. For example, the DM can learn about the nature of the trade-offs among the objectives of the problem (e.g., disconnectedness, convexity, knees) or discover inconsistencies of the model with respect to the real problem.

Nonetheless, approximating the whole Pareto front might not be convenient in all cases. For instance, in problems with expensive function evaluations, introducing preference information during the search can avoid the waste of function evaluations in regions in which the DM is definitively not interested. Additionally, many real world problems aim at improving an existing design which can be used as a reference design point. The incorporation of preferences can also help to deal with multiobjective optimization problems with a high number of objectives (many-objective problems). One of the difficulties of these problems is that the number of points required to accurately represent a Pareto front increases exponentially with the number of objectives [5]. This introduces a challenge for the decision maker, since the selection of one solution from among a huge number of solutions is evidently a very difficult task.

In this paper, we evaluate the performance of a previously proposed interactive method [6], which is applied to a real world problem. The main component of the interactive method is a preference relation based on an achievement scalarizing function [7]. Regarding the real world application, we adopt three aerodynamic airfoil shape optimization problems. As we will see later on, the experimental results show that, in general, the interactive method achieves better results than a traditional *a posteriori approach*. However, in our experiments, we found out that, when using an aerodynamic problem with 6 objectives, only 3 of them were improved. We hypothesize that this was due to the presence of several local Pareto fronts in that problem. In order to validate our hypothesis, we carried out some additional experiments to analyze the effect of local Pareto fronts in the convergence of the interactive method adopted in our work.

The remainder of this paper is organized as follows. The next section presents some basic concepts and the notation adopted throughout the paper. Section III introduces a new preference relation to incorporate preferences and its integration in an interactive method. Section IV presents the experimental results of the interactive method to solve three airfoil shape optimization problems. Finally, in Section V we present our conclusions and some potential paths for future research.

II. Basic Concepts and Notation

In this section, we will introduce the concepts and notation used throughout the rest of the paper.

A. Multiobjective Optimization Problems

Definition 1: A MOP is defined as:

$$\begin{array}{ll} \text{Minimize} & \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})]^T \\ \text{subject to} & \mathbf{x} \in \mathcal{X}. \end{array} \tag{1}$$

The vector $\mathbf{x} \in \mathbb{R}^n$ is formed by n *decision variables* representing the quantities for which values are to be chosen in the optimization problem. The *feasible set* $\mathcal{X} \subseteq \mathbb{R}^n$ is implicitly determined by a set of equality and inequality constraints. The vector function $\mathbf{f} : \mathcal{X} \rightarrow \mathbb{R}^k$ is composed by $k \geq 2$ scalar *objective functions* $f_i : \mathcal{X} \rightarrow \mathbb{R}$ $(i = 1, \ldots, k)$. In multiobjective optimization, the sets $\mathbb{R}^n$ and $\mathbb{R}^k$ are known as *decision variable space* and *objective function space*, respectively. The image of $\mathcal{X}$ under the function $\mathbf{f}$ is a subset of the objective function space denoted by $\mathcal{Z} = \mathbf{f}(\mathcal{X})$ and referred to as the *feasible set in the objective function space*.

In $\mathbb{R}^k$ there is no canonical order, and thus, we need weaker definitions of order to compare vectors. In multiobjective optimization, the *Pareto dominance relation* is usually adopted to compare vectors in $\mathbb{R}^k$.

Definition 2: We say that a vector $\mathbf{z}^1$ dominates vector $\mathbf{z}^2$, denoted by $\mathbf{z}^1 \prec_{\text{par}} \mathbf{z}^2$, if and only if:

$$\forall i \in \{1, \ldots, k\} : z_i^1 \leq z_i^2 \text{ and } \exists i \in \{1, \ldots, k\} : z_i^1 < z_i^2.$$

Definition 3: A solution $\mathbf{x}^* \in \mathcal{X}$ is Pareto optimal if there does not exist another solution $\mathbf{x} \in \mathcal{X}$ such that $\mathbf{f}(\mathbf{x}) \prec_{\text{par}} \mathbf{f}(\mathbf{x}^*)$.

Definition 4: The Pareto optimal set, P_{opt}, is defined as:

$$P_{\text{opt}} = \{\mathbf{x} \in \mathcal{X} \mid \nexists\, \mathbf{y} \in \mathcal{X} : \mathbf{f}(\mathbf{y}) \prec_{\text{par}} \mathbf{f}(\mathbf{x})\}. \tag{2}$$

Definition 5: For a Pareto optimal set, P_{opt}, the Pareto front, PF_{opt}, is defined as:

$$PF_{\text{opt}} = \{\mathbf{z} = (f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})) \mid \mathbf{x} \in P_{\text{opt}}\}. \tag{3}$$

The goal of *a posteriori* MOEAs is to find an approximation set, PF_{approx}, of the Pareto optimal front.

In interactive optimization methods it is useful to know the lower and upper bounds of the Pareto front. The *ideal point* represents the lower bound and is defined by $z_i^\star = \min_{z \in \mathcal{Z}}(z_i)$ for all $i = 1, \ldots, k$. In turn, the upper bound is defined by the *nadir point*, which is given by $z_i^{\text{nad}} = \max_{z \in PF_{\text{opt}}}(z_i)$ for all $i = 1, \ldots, k$. Another useful point is one strictly better than $z_i^\star$, i.e., the one called *utopian point*, defined by $z_i^{\star\star} = z_i^\star - \epsilon\ \forall i = 1, \ldots, k$, where $\epsilon > 0$ is a small scalar.

B. Achievement Scalarizing Functions

The proposed preference relation is based on the achievement scalarizing function approach proposed by Wierzbicki [7]. An achievement scalarizing function uses a reference point to capture DM's preference information in the form of desired values for each objective function.

Definition 6: An achievement scalarizing function (or achievement function for short) is a parameterized function $s(\mathbf{z}, \mathbf{z}^{\text{ref}}) : \mathbb{R}^k \rightarrow \mathbb{R}$, where $\mathbf{z}^{\text{ref}} \in \mathbb{R}^k$ is a reference point representing the decision maker's aspiration levels.

Thus, the multiobjective problem is transformed into $\min_{\mathbf{z} \in \mathcal{Z}} s(\mathbf{z}, \mathbf{z}^{\text{ref}})$. A common achievement function, based on the Chebyshev distance [8], [9], is the weighted achievement function.

Definition 7: The weighted achievement function is defined by

$$s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}}) = \max_{i=1,\ldots,k}\{\lambda_i(z_i - z_i^{\text{ref}})\} + \rho \sum_{i=1}^{k} \lambda_i(z_i - z_i^{\text{ref}}), \tag{4}$$

where $\mathbf{z}^{\text{ref}}$ is a reference point, $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_k]$ is a vector of weights such that $\forall i\ \lambda_i \geq 0$ and, for at least one i, $\lambda_i > 0$, and $\rho > 0$ is an augmentation coefficient sufficiently small. We should note that, unlike the Chebyshev distance, the achievement function does not use the absolute value in the first term. This small difference allows the achievement function to correctly assess solutions that improve the reference point.

In most reference point methods, the weight vector, $\boldsymbol{\lambda}$, does not define preferences, but is mainly used for normalizing each objective function [10]. Usually, the weights are set as

$$\lambda_i = \frac{1}{z_i^{\text{nad}} - z_i^{\star\star}}, \text{ for all } i = 1, \ldots, k. \tag{5}$$

It is important to mention that the DM can provide both feasible and infeasible reference points, or more precisely, $\mathbf{z}^{\text{ref}} \in \mathcal{Z} + \mathbb{R}_+^k$ or $\mathbf{z}^{\text{ref}} \notin \mathcal{Z} + \mathbb{R}_+^k$, where $\mathbb{R}_+^k$ is the nonnegative orthant of $\mathbb{R}^k$. On the one hand, if $\mathbf{z}^{\text{ref}} \in \mathcal{Z} + \mathbb{R}_+^k$, then the minimization of Eq. 4 subject to $\mathbf{z} \in \mathcal{Z}$ should represent the maximization of the surplus $\mathbf{z} - \mathbf{z}^{\text{ref}} \in \mathbb{R}^k$. On the other hand, if $\mathbf{z}^{\text{ref}} \notin \mathcal{Z} + \mathbb{R}_+^k$, the minimization of Eq. 4 subject to $\mathbf{z} \in \mathcal{Z}$ minimizes the distance between the reference point and the Pareto optimal set.

III. Guiding the Search Using an Achievement Function

In this section, we present the Chebyshev preference relation proposed in [6]. This preference relation, based on the Chebyshev achievement function (see Eq. 4), provides a simple way to integrate preferences into different types of MOEAs. Further, we show an interactive procedure for multi- and many-objective optimization problems in order to redefine a Region of Interest (ROI) iteratively, and eventually select a single preferred solution. Similar approaches to the one proposed here can be found in the specialized literature (see e.g., [11], [12], [13]). In a previous work [6] we discuss the differences of some of those approaches with respect to the Chebyshev preference relation.

A. The Chebyshev Preference Relation

The basic idea of the Chebyshev preference relation is to combine the Pareto dominance relation and an achievement function to compare solutions in objective function space. First, the achievement function value, $s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$, is computed for each solution $\mathbf{z}$. Then, the objective space is divided into two regions. One region defines the Region of Interest and contains those solutions with an achievement value less or equal to $s^{\min} + \delta$, where $s^{\min} = \min_{\mathbf{z}\in\mathcal{Z}} s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$, and δ is a threshold that determines the size of the ROI. Fig. 1 shows the ROI defined by means of the achievement function. Solutions in this region are compared using the usual Pareto dominance relation, while solutions outside of the ROI are compared using their achievement function value.

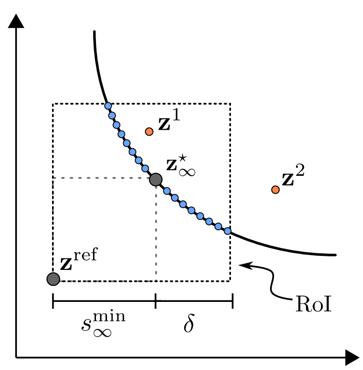

Fig. 1. Nondominated solutions with respect to the Chebyshev relation.

Formally, the Chebyshev preference relation is defined as follows.

Definition 8: A solution $\mathbf{z}^1$ is preferred to solution $\mathbf{z}^2$ with respect to the Chebyshev relation ($\mathbf{z}^1 \prec_{\text{cheby}} \mathbf{z}^2$), if and only if:

1) $s_\infty(\mathbf{z}^1, \mathbf{z}^{\text{ref}}) < s_\infty(\mathbf{z}^2, \mathbf{z}^{\text{ref}}) \;\wedge$
$\{\mathbf{z}^1 \notin R(\mathbf{z}^{\text{ref}}, \delta) \vee \mathbf{z}^2 \notin R(\mathbf{z}^{\text{ref}}, \delta)\}$, or,

2) $\mathbf{z}^1 \preceq_{\text{par}} \mathbf{z}^2 \wedge \{\mathbf{z}^1, \mathbf{z}^2 \in R(\mathbf{z}^{\text{ref}}, \delta)\}$,

where $R(\mathbf{z}^{\text{ref}}, \delta) = \{\mathbf{z} \,|\, s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}}) \leq s^{\min} + \delta\}$ is the Region of Interest with respect to the vector of aspiration levels $\mathbf{z}^{\text{ref}}$.

As an illustration of the preference relation, consider solutions $\mathbf{z}^1$ and $\mathbf{z}^2$ presented in Fig. 1. Since $\mathbf{z}^2 \notin R(\mathbf{z}^{\text{ref}}, \delta)$ and $s_\infty(\mathbf{z}^1, \mathbf{z}^{\text{ref}}) < s_\infty(\mathbf{z}^2, \mathbf{z}^{\text{ref}})$, then $\mathbf{z}^1 \prec_{\text{cheby}} \mathbf{z}^2$.

Unlike some distance metrics, the achievement function (Eq. 4) allows a MOEA to find points in problems with nonconvex Pareto fronts. In order to incorporate the Chebyshev relation into the two previously mentioned MOEAs we only have to change the usual Pareto dominance checking procedure by the function that implements the new relation.

The threshold δ can be adjusted in terms of the proportion of the current range of the achievement function (i.e., the difference between the minimum and maximum achievement with respect to a given solution set P). If $\tau \in [0, 1]$ is that proportion, then $\delta = \tau \cdot (s^{\max} - s^{\min})$, where $s^{\max} = \max_{\mathbf{z}\in P} s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$ and $s^{\min} = \min_{\mathbf{z}\in P} s_\infty(\mathbf{z}, \mathbf{z}^{\text{ref}})$.

In [6] was also proposed a variant of the Chebyshev relation that uses an approximation of the ideal point as reference point in definition 8. This variant is called the *central-guided Chebyshev relation* since it focuses the search towards the ideal point.

B. An Interactive Method Using the Chebyshev Relation

When the DM does not have enough knowledge about the problem to provide a reference point, the central-guided Chebyshev relation can be used to obtain a first set of solutions. However, having a previous best known solution of the given problem is common in real-world problems. In that case, the previous solution can serve as a good reference point. Then, the process can follow the usual steps of the interactive techniques. That is, at each iteration the DM must provide new aspiration levels in the form of a reference point. Additionally, the DM can change the value of the threshold τ to control the size of the set of solutions. For example, the user can set $\tau = 0.5$ in order to obtain about half of the Pareto front around the reference point. In order to ease the visualization of the solutions, a technique for truncating the approximation set can be used. For example, a clustering technique can be employed, such as the one used in Strength Pareto Evolutionary Algorithm 2 (SPEA2) [3], or a technique similar to the archiving methods. Therefore, the interactive process requires an additional parameter indicating the number of solutions to visualize. This interactive process continues until the DM is satisfied with a solution of the current set of solutions. Algorithm 1 shows the whole interactive process.

IV. Experimental Study

A. Airfoil Shape Problem with 2 Objectives

In order to illustrate the interactive method presented in the previous section we will use a multiobjective aerodynamic airfoil shape optimization problem adapted from [14], and having 2 objectives. The goal is to optimize the shape of a standard-class glider, aiming at obtaining optimum performance for a sailplane.

1) Objective functions: Two conflicting objective functions are defined in terms of a sailplane average weight and operating conditions [14]:

1) Min $f_1 = C_D/C_L$,
s.t. $C_L = 0.63, Re = 2.04 \times 10^6,\ M = 0.12$.
2) Min $f_2 = C_D/C_L^{3/2}$,
s.t. $C_L = 1.05, Re = 1.29 \times 10^6, M = 0.08$.

Objective f_1 represents the inverse of the glider's gliding ratio, whereas f_2 represents the sink rate. Both objectives are important performance measures for this aerodynamic optimization problem. C_D and C_L are the drag and lift coefficients. Each objective is evaluated at different prescribed flight conditions, given in terms of Mach and Reynolds numbers. The aim of solving this MOP, is to find a better airfoil shape, which improves a reference design.

2) Geometry parameterization: In the present case study, a modified PARSEC airfoil representation [15] is used. Fig. 2 illustrates the 12 basic parameters used for this representation: $r_{\text{le}_{\text{up}}}$ / $r_{\text{le}_{\text{lo}}}$ leading edge radius for upper/lower surfaces, $X_{\text{up}}/X_{\text{lo}}$ location of maximum thickness for upper/lower surfaces, $Z_{\text{up}}/Z_{\text{lo}}$ maximum thickness for upper/lower surfaces, $Z_{xx\text{up}}/Z_{xx\text{lo}}$ curvature for upper/lower surfaces, at maximum thickness locations, Z_{te} trailing edge coordinate, ΔZ_{te} trailing edge thickness, α_{te} trailing edge direction, and β_{te} trailing edge wedge angle. The PARSEC geometry representation adopted allows us to define independently the leading edge radius, both for upper and lower surfaces (the original representation uses the same value both for upper and lower surfaces). Thus, 12 variables are used in total. Their allowable ranges are defined in Table I.

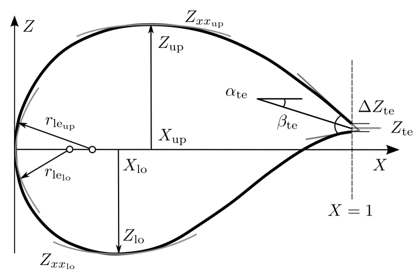

Fig. 2. PARSEC airfoil parametrization.

The PARSEC airfoil geometry representation uses a linear combination of shape functions for defining the upper and lower surfaces. These linear combinations are given by:

$$Z_{\text{upper}} = \sum_{n=1}^{6} a_n x^{(n-1)/2}, \quad Z_{\text{lower}} = \sum_{n=1}^{6} b_n x^{(n-1)/2} \qquad (6)$$

The coefficients a_n, and b_n are determined as function of the 12 geometric parameters by solving two systems of linear equations, one for each surface. It is important to note that the geometric parameters $r_{\text{le}_{\text{up}}}/r_{\text{le}_{\text{lo}}}$, $X_{\text{up}}/X_{\text{lo}}$, $Z_{\text{up}}/Z_{\text{lo}}$, $Z_{xx_{\text{up}}}/Z_{xx_{\text{lo}}}$, Z_{te}, ΔZ_{te}, α_{te}, and β_{te} are the actual design variables in the optimization process. In turn, coefficients a_n, b_n serve as intermediate variables for interpolating the airfoil's coordinates, which are used by the Computational Fluid Dynamics (CFD) solver (we used the Xfoil CFD code [16]) for its discretization process.

TABLE I

PARAMETER RANGES FOR THE PARSEC AIRFOIL REPRESENTATION FOR PROBLEMS A720 (2 AND 3 OBJS.) AND NLF0416 (6 OBJS.).

	A720		**NLF0416**	
Variable	**Lower**	**Upper**	**Lower**	**Upper**
$r_{\text{le}_{\text{up}}}$	0.0085	0.0126	0.0055	0.0215
$r_{\text{le}_{\text{lo}}}$	0.0020	0.0040	0.0055	0.0215
α_{te}	7.0000	10.0000	-2.0000	21.0000
β_{te}	10.0000	14.0000	1.0000	15.0000
Z_{te}	-0.0060	-0.0030	-0.0200	0.0200
ΔZ_{te}	0.0025	0.0050	0.0000	0.0000
X_{up}	0.4100	0.4600	0.2875	0.5345
Z_{up}	0.1100	0.1300	0.0880	0.1195
$Z_{xx_{\text{up}}}$	-0.9000	-0.7000	-1.0300	-0.4200
X_{lo}	0.2000	0.2600	0.3060	0.5075
Z_{lo}	-0.0230	-0.0150	-0.0650	-0.0500
$Z_{xx_{\text{lo}}}$	0.0500	0.2000	-0.0490	0.8205

Algorithm 1 Interactive technique using the Chebyshev preference relation.

Step 1: Ask the DM to specify the threshold τ. If the DM has some knowledge about the problem, he/she can provide a reference point. Otherwise, the central-guided preference relation can be used to converge towards the ideal point.

Step 2: **If** a reference point was provided, **then**
 Execute the MOEA using the Chebyshev relation with the reference point provided by the decision maker.
else
 Execute the MOEA using the central-guided Chebyshev relation.

Step 3: Ask the DM to define how many solutions of the current approximation should be shown. Additionally, from the use of the central-guided relation the DM can be informed of the current ideal point in order to decide the new aspiration levels.

Step 4: **If** the DM is satisfied with some solution of the current set, **then**
 STOP.
else
 Go to **Step 1**.

Next, we will show a simulation of the interactive process using Nondominated Sorting Genetic Algorithm II (NSGA-II) with the Chebyshev relation based on a reference point. We adopted the following parameters for NSGA-II: a crossover probability of 0.9, a mutation probability of $1/n$ (n is the number of decision variables), and the distribution indices for crossover and mutation were set as 15 and 20, respectively. A population composed of 60 individuals was employed.

In all the experiments included in the paper we used $\rho = 10^{-5}$ for Eq. 4. In the first step of the process, we used $\tau = 0.8$ in order to get a global perspective of the entire Pareto front. As a reference point we employed the vector $\mathbf{z}^{\text{ref}} = [0.007610, 0.005236]$. This reference point corresponds to the evaluation of a reference airfoil shape A720 [14] in both objectives. Then, NSGA-II was executed for 15 generations. The resulting approximation set is shown in Fig. 3 (denoted by triangles). As can be seen, the reference point was dominated by almost all solutions in the approximation set. This illustrates how the relation is able to correctly compare solutions better than the reference point provided. On the other hand, due the nature of the objective space of the problem, only 25 solutions, from the total of 60, are nondominated. Therefore, in this case, the clustering technique to reduce the size of the approximation set was not needed.

Since the initial reference point was improved, we decided

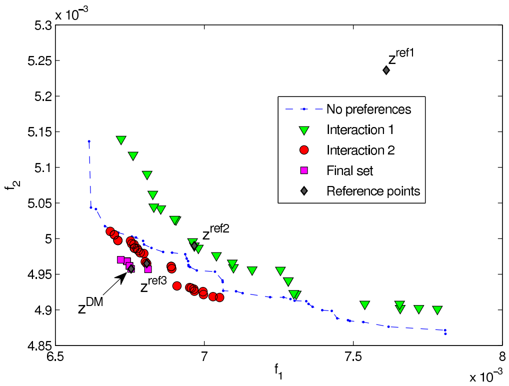

Fig. 3. Simulation of the interactive method.

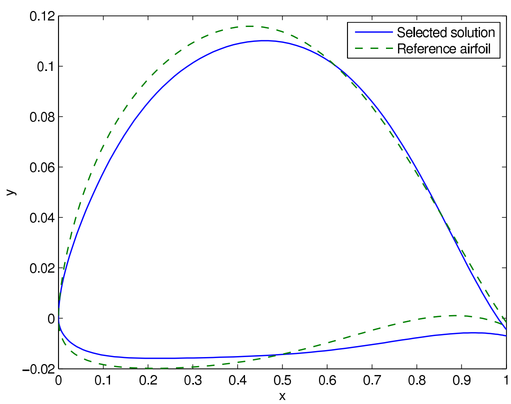

Fig. 4. Airfoil of the most preferred solution from the simulation of the interactive method.

to choose one solution of the approximation set as the next reference point, namely, the nearest solution to the ideal point (diamond). For the next execution, the region of interest was reduced to $\tau = 0.2$. Similar to the previous DM interaction, the next reference point was the nearest solution of PF_{approx} to the ideal point. In order to obtain a final approximation to select the most preferred solution, the region of interest was reduced to a small region using $\tau = 0.05$. This time NSGA-II was executed for 40 generations. At this stage only 8 solutions were obtained and the most preferred solution for the DM was the one with objective values $[0.006754, 0.004957]$. Fig. 4 shows the airfoils corresponding to the initial reference point and to the most preferred solution. In this example, an improvement of approximately 11.24% and of 5.32% was attained for the first and second objective, respectively. From a practical point of view, these improvements are quite significant in increasing the aerodynamic efficiency of the sailplane.

Fig. 3 also shows the PF_{approx} achieved by NSGA-II with no preferences during the same number of generations than that used in the interactive method. As one can expect, the final approximation set obtained articulating preferences is closer to the ideal point than the one generated with no preferences. This can be explained by the fact that the incorporation of preferences concentrates all the function evaluations to improve the region of interest. On the other hand, when the task is to approximate the entire Pareto front, some function evaluations are used to approximate regions outside the region of interest. These are clearly different tasks, and therefore, a fair performance comparison is not possible. Nonetheless, we want to emphasize the computational savings of using an interactive approach over an *a posteriori* approach, specially when the function evaluations are expensive in terms of CPU time.

B. Airfoil Shape Problem with 3 Objectives

Here, we will evaluate the interactive method using two airfoil shape optimization problems with 3 and 6 objectives, respectively. This time, we will simulate the DM using the Chebyshev achievement function. Specifically, at each interaction point, the new reference point will be the solution in the current PF_{approx} with the best achievement value (which is to be minimized). For the simulation of the 3-objective problem we used 4 interaction points with the DM during the search, and for the 6-objective problem we used 3 interaction points. The parameters at each interaction point are shown in Table II. The initial threshold for both problems was set to $\tau = 0.8$.

TABLE II
PARAMETER VALUES AT EACH INTERACTION POINT.

Problem		**Int. 1**	**Int. 2**	**Int. 3**	**Int. 4**
3-obj	Gen	15	35	55	80
	τ	0.5	0.2	0.1	0.025
6-obj	Gen	15	35	55	–
	τ	0.43	0.18	0.025	–

In order to evaluate the performance of the interactive method, for each run, the best achievement value of the final PF_{approx} was measured. As a reference, we also computed the best achievement value obtained by NSGA-II with no preferences. The 3-objective problem is a variant of the problem A720 in which the first and third objectives are objectives f_1 and f_2 of the 2-objective problem of the previous section. The second objective is defined as

- Min $f_2 = C_D/C_L$,
  s.t. $C_L = 0.86$, $Re = 1.63 \times 10^6$, $M = 0.1$.

The bounds for the variables are the same described in Table I. For this problem, we used the vector $[0.007610, 0.005895, 0.005236]$ as our initial reference point. The results for the 3-objective problem are shown in Table III. As can be seen, both approaches yield achievement values results less than zero, which means that the reference point was improved in all cases. In addition, as expected, the interactive approach obtained better results than the approach with no preferences articulated. The solution with the best achievement value was $[0.006772, 0.005244, 0.004960]$. Objectives were improved by 11.01%, 11.04% and 5.27%, respectively. The airfoil of this solution is presented in Fig. 5, along with that of the reference point.

C. Airfoil Shape Problem with 6 Objectives

The 6-objective problem was taken from [17]. The goal of this problem is to optimize the airfoil shape of a low-speed unmanned aerial vehicle to cover a range of different flight

TABLE III

STATISTICS OF THE ACHIEVEMENT FUNCTION VALUES OBTAINED WITH PREFERENCES AND WITHOUT THEM IN THE 3-OBJECTIVE PROBLEM.

	Best	**Median**	**Worst**	**Std. dev.**
Preferences	-0.2196	-0.2111	-0.1982	0.0047
No prefs.	-0.2183	-0.2020	-0.1816	0.0101

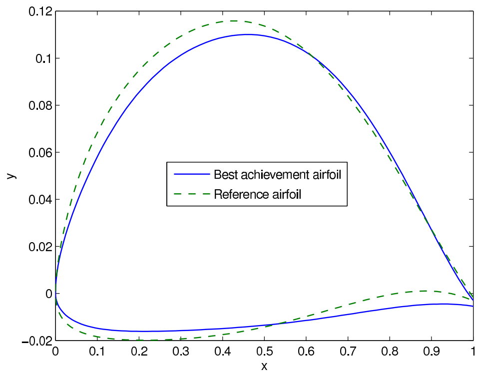

Fig. 5. Airfoil with the best achievement value and the reference airfoil for the problem with 3 objectives.

TABLE V

STATISTICS OF THE ACHIEVEMENT FUNCTION VALUES OBTAINED WITH PREFERENCES AND WITHOUT THEM IN THE 6-OBJECTIVE PROBLEM.

	Best	**Median**	**Worst**	**Std. dev.**
Preferences	0.0047	0.0473	0.0914	0.0183
No prefs.	0.0157	0.2506	0.4787	0.1480

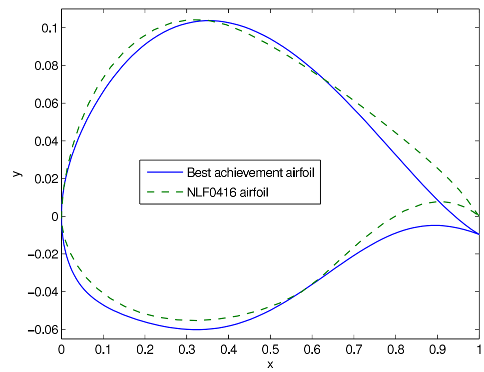

Fig. 6. Airfoil with the best achievement value and the reference airfoil for the problem with 6 objectives.

condition (e.g., take-off and cruise). The 6 objectives to be minimized are described in Table IV, and the bounds for the variables are presented in Table I.

As a reference point we employed a representative profile of the NLF series, namely the NLF0416 [18], $\mathbf{z}^{\text{ref}} = [0.00523, 0.00595, 0.01048, 0.33373, 0.90135, 2.93083]$.

The results presented in Table V show that for this problem the reference point was not improved by any of the two approaches. However, the interactive approach found better airfoils than the those of the approach without preferences. The solution corresponding with the best achievement value found by the interactive approach is the following: $[0.004962, 0.007022, 0.007275, 0.346273, 0.920056, 2.929393]$. This solution improves objectives f_1, f_3 and f_6 by an amount of 5.12%, 30.58% and 0.04%, respectively. The airfoil of this solution is presented in Fig. 6. Since this problem has local Pareto fronts, we believe that this feature avoids improving the reference point. For this reason, in the next section we analyze the relation of the convergence and the size of the ROI in the presence of several local Pareto fronts.

D. Impact of Local Pareto Fronts on Convergence

In this section we investigate the effect of the size of the ROI on the convergence of a MOEA. We study two different

TABLE IV

OBJECTIVES OF THE AIRFOIL DESIGN PROBLEM WITH 6 OBJECTIVES.

Objective	**Comments**
$f_1 = C_d$	$C_l = 0.5, \text{Re} = 4 \times 10^6, \text{Ma} = 0.3$
$f_2 = C_d / C_l^{3/2}$	$\text{Re} = 4 \times 10^6, \text{Ma} = 0.3$
$f_3 = C_{m_0}^2$	$\text{Re} = 4 \times 10^6, \text{Ma} = 0.3$
$f_4 = 1/C_{l_{\max}}^2$	$\text{Re} = 4 \times 10^6, \text{Ma} = 0.3$
$f_5 = 1/C_l^2$	$\alpha = 5°, \text{Re} = 2 \times 10^6, \text{Ma} = 0.15$
$f_6 = 1/x_{tr}$	$\alpha = 5°, \text{Re} = 2 \times 10^6, \text{Ma} = 0.15$

scenarios, one with several local Pareto fronts, and another with only the global Pareto front. The natural hypothesis is that the convergence of a MOEA will be improved as the ROI becomes smaller since the resources are always concentrated in a smaller region. Nonetheless, in problems with many local Pareto fronts the improvement in convergence is not so evident. The reasoning is that a search in a constrained region is also less diverse, and, as a result, the MOEA might be more prone to get stuck in one of the local Pareto fronts. To carry out this analysis we employed the problems DTLZ2 and DTLZ3. The difference between these two problems is that the latter has $(3^\ell - 1)$ local Pareto fronts, where ℓ is the number of distance-related variables[1]. In these experiments the size of the ROI was varied according to $\tau = 1.0, 0.4, 0.2, 0.025$. It is worth noting that the original NSGA-II is obtained when $\tau = 1.0$. For each problem, we used 3, 6 and 9 objectives. For both MOPs we used $\ell = 10$. In order to assess convergence, we adopted the generational distance (GD). For both DTLZ problems we used the exact GD, namely $GD = \frac{1}{m} \sum_{\mathbf{z} \in PF} \sum_{j=1}^{M} (z_j)^2 - 1$, where $m = |PF|$. The results presented are the average over 50 runs of each NSGA-II configuration.

Regarding the problem DTLZ2 we can clearly see in Fig. 7 that the generational distance is greatly improved when the region of interest is small. Furthermore, as we expect, the best GD value is obtained when $\tau = 0.025$. Another interesting result is the deterioration of NSGA-II's search ability when the number of objectives is increased. As pointed out elsewhere, the reason of that deterioration is the generation of dominance resistant solutions coupled with some mechanism of NSGA-II. For instance, since dominance resistant solution are distributed along large areas of the objective function space, they receive much better crowding values than nondominated solutions

[1]Distance-related variables control the progress towards the Pareto optimal front.

near the Pareto front. Thus, dominance resistant solutions are ranked in the first nondominated layer.

The results of GD at the last generation (see Fig. 8) reveal that NSGA-II with $\tau < 1$ was able to converge very close to the Pareto front even with 9 objectives. This means that the incorporation of preferences also helps to overcome the difficulties posed by many-objective problems.

With respect to DTLZ3, we can observe in Fig. 9 that NSGA-II with $\tau < 1$ still maintains a tendency to converge. However, the local Pareto fronts causes that none of the three configurations with $\tau < 1$ show a clear advantage over the others during the first half of the search. A possible explanation is that for $\tau < 1$, NSGA-II gets trapped in a local optimum regardless of the size of the region of interest. In fact, for 3 and 6 objectives (see Fig. 9) the configuration with $\tau = 0.2$ converges faster than the one with $\tau = 0.025$. In contrast, after the first half of the search, the three configurations show a convergence similar to that of DTLZ2. That behavior can be explained by the fact that once the local Pareto fronts have been surpassed, NSGA-II can approach the global Pareto front as in DTLZ2. The last local Pareto front of DTLZ3 is an sphere of radius 2, i.e., when $GD= 1$, which perfectly matches the point at which the convergence behavior changes (see the dashed line in Fig. 9). The presence of outliers in the boxplots (denoted by crosses above the boxplots) shown in Fig. 10, and the height of the boxes reveal a wide variation of GD. This is due to the fact that in some runs, the NSGA-II was not able to overcome the local Pareto fronts before the last generation, or it did it, but until reaching the end of the search.

V. Conclusions and Future Work

In this paper, we applied an interactive optimization technique to solve three aerodynamic airfoil shape optimization problems in order to assess its performance in real world problems. The interactive technique is based on a preference relation that incorporates the preferences of the DM by means of a reference point. In order to evaluate the performance of the interactive optimization technique we adopted NSGA-II as our search engine and compared its outcome using the interactive technique (with preferences) against those using the original *a posteriori* approach (i.e., no preferences). First, in the three airfoil shape optimization problems considered in this study, the most preferred solution chosen by the DM using the interactive method yields a better achievement value than in the analogous case using the *a posteriori* approach. Additionally, the reference points used in the problems with 2 and 3 objectives were dominated by the most preferred solution attained. However, in the problem with 6 objectives only three objectives of the reference solution were improved by the solution selected by the DM. Since we suspected that this result was due to the presence of local optima in the problem, we carried out an analysis using two benchmark problems, namely, one with several local Pareto fronts, and another one with the Pareto optimal front only. The experimental results reveal that in the problem with only one local Pareto front, a decrease in the size of the ROI always implies an improvement in convergence (in terms of generational distance). In contrast, in the problem with several local Pareto fronts, the convergence does not always improve when the ROI is reduced. As the ROI gets smaller, at some point, the convergence does not improve anymore. In fact, in some cases, a marginally better convergence was observed using a ROI of moderate size instead of a very small one. As future work we want to apply the interactive method in a MOEA that uses different evolutionary operators, for example, those of differential evolution.

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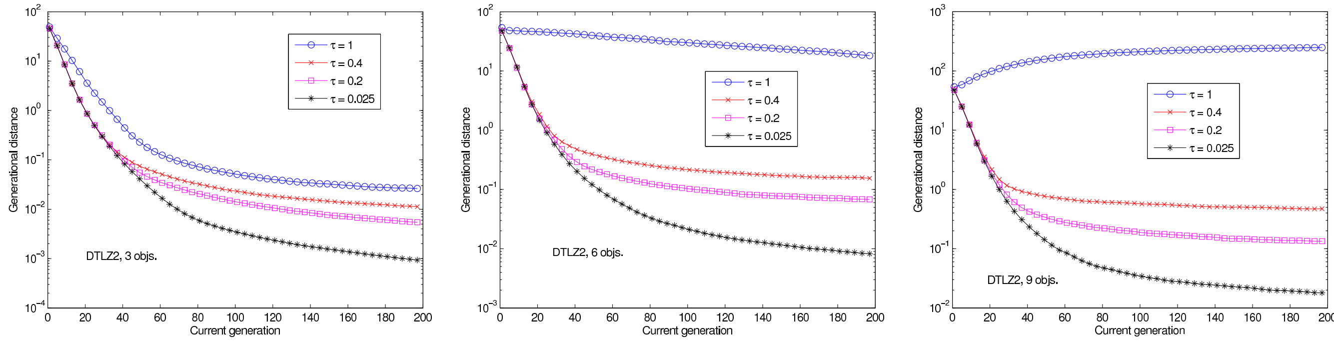

Fig. 7. Effect of the size of the region of interest on convergence during the search. Note that $\tau = 1$ is equivalent to the usual NSGA-II.

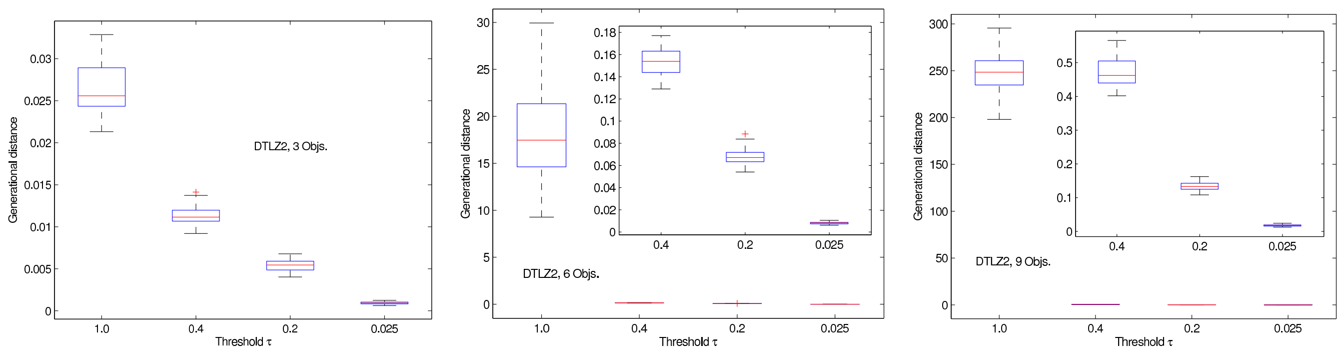

Fig. 8. Effect of the size of the region of interest on convergence. Note that $\tau = 1$ is equivalent to the usual NSGA-II.

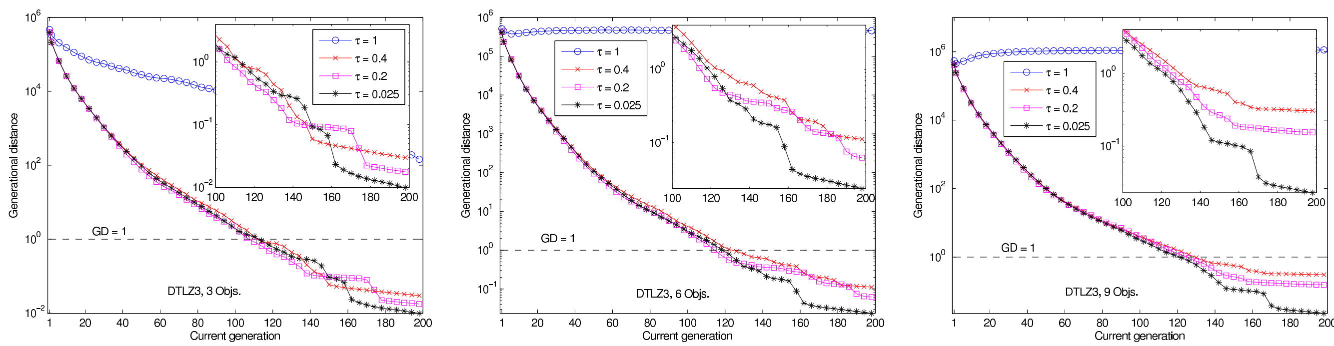

Fig. 9. Effect of the size of the region of interest on convergence during the search. Note that $\tau = 1$ is equivalent to the usual NSGA-II.

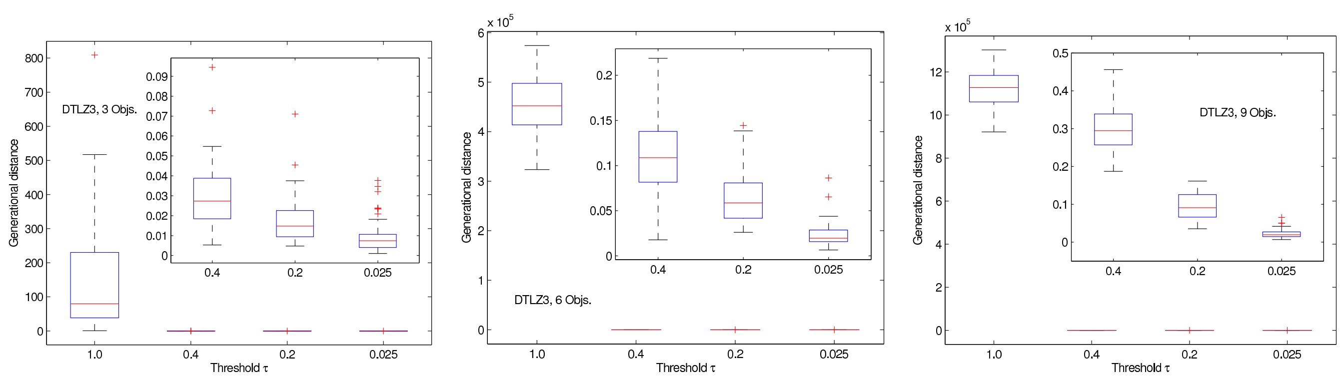

Fig. 10. Effect of the size of the region of interest on convergence. Note that $\tau = 1$ is equivalent to the usual NSGA-II.

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