Focus Article

Evolutionary multiobjective optimization

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This paper presents a very short introduction to multiobjective evolutionary algorithms, including their basic concepts and their main components. The discussion focuses on algorthmic design and, therefore, the issues discussed include selection mechanisms, diversity maintenance mechanisms, and elitism in a multiobjective context. © 2011 John Wiley & Sons, Inc. *WIREs Data Mining Knowl Discov* 2011 1 444–447
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INTRODUCTION

In the real world, many problems have two or more (conflicting) objectives which we would like to optimize at the same time. The solution of these *multiobjective optimization problems* (MOPs) has raised a lot of interest within operations research during the last 35 years.[1] However, and in spite of the relatively large number of mathematical programming approaches currently available for solving MOPs, their limitations (related, e.g., to the specific features of the problem being solved) have motivated the development of alternative techniques such as the metaheuristics[a] from which evolutionary algorithms (EAs) are, with no doubt, the most popular.[3]

The first implementation of a multiobjective evolutionary algorithm (MOEA) dates back to 1985.[4] However, this area, which is now called 'evolutionary multiobjective optimization', or EMO, has experienced a very important growth, mainly in the last 15 years.[3b]

BASIC CONCEPTS

MOPs are problems of the type[c]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})]^T \tag{1}$$

subject to

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m, \tag{2}$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p, \tag{3}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}, i = 1, 2, \ldots, k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}, i = 1, \ldots, m, j = 1, 2, \ldots, p$ are the constraint functions of the problem.

Definition 1. Given two vectors $\vec{x}, \vec{y} \in \mathbb{R}^k$, we say that $\vec{x} \leq \vec{y}$ if $x_i \leq y_i$ for $i = 1, \ldots, k$, and that $\vec{x}$ *dominates* $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

Definition 2. We say that a vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is *nondominated* with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. We say that a vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is *Pareto optimal* if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The *Pareto Optimal Set* $\mathcal{P}^*$ is defined by

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto optimal}\}.$$

Definition 5. The *Pareto Front* $\mathcal{PF}^*$ is defined by

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}.$$

The aim is thus to determine the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3). Note, however, that in practice, not all the Pareto optimal set is normally desirable or achievable.

MULTIOBJECTIVE EVOLUTIONARY ALGORITHMS

EAs offer two main advantages with respect to mathematical programming techniques, when dealing with MOPs: (1) since they rely on the use of a set of solutions at each iteration, they can find several

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elements of the Pareto optimal set in a single run, instead of only one at a time; and (2) EAs tend are normally less susceptible to the shape or continuity of the Pareto front than mathematical programming techniques.

MOEAs extend a traditional EA in two main aspects:

1 *Selection mechanism*: In MOEAs, the aim is to select nondominated solutions and not the solutions with the highest fitness. In addition, and according to the definition of Pareto optimality, all the nondominated solutions in a population are normally considered as equally good.
2 *Diversity maintenance*: MOEAs require a mechanism that preserves diversity and avoids convergence to a single solution (this will eventually happen because of stochastic noise, if an EA is run for a sufficiently long time).

Regarding selection, there are several possible mechanisms that can be used to solve MOPs:

- *Aggregating functions*: In this case, the objectives are normally combined in some form (using either linear or nonlinear schemes), such that a single (scalar) value is generated. This scalar value is adopted as the fitness value of the EA. These approaches were very popular in the early days of MOEAs (particularly, linear aggregating functions).[3] Today, the use of nonlinear aggregating functions that provide a ranking of solutions has become popular again because they seem to work better than Pareto ranking in problems having more than three objectives.[5] An interesting type of aggregating approach is the so-called *scalarization*, in which a MOP is transformed into several single-objective optimization problems. This sort of approach has been adopted by several MOEAs (e.g., see Ref 6). However, the most popular of these approaches is MOEA/D,[7] in which the optimization of the scalar subproblems generated by a decomposition approach is done in a very efficient way.
- *Pareto-based selection*: The most popular scheme within this group is called Pareto ranking, and its main idea is to sort the population of an EA based on Pareto dominance, such that all nondominated individuals are assigned the same rank (or importance). The aim is that all nondominated individuals get the same probability of being selected, and that such probability is higher than the one corresponding to individuals which are dominated. Although conceptually simple, this sort of selection mechanism allows for a wide variety of possible implementations.[3] That is the reason why several MOEAs based on Pareto ranking have been proposed (e.g., SPEA[8] and NPGA [9]). From them, the Nondominated Sorting Genetic Algorithm-II (NSGA-II)[10] remains as the most popular in the current literature.
- *Indicator-based selection*: The idea in this case is to adopt a performance measure to select solutions. This concept attracted attention when the *Indicator-Based Evolutionary Algorithm* (IBEA) was proposed.[11] Within a similar line of thought, but without explicitly considering the incorporation of user's preferences (as in IBEA), the *S Metric Selection Evolutionary Multiobjective Optimization Algorithm* (SMS-EMOA)[12] adopts a selection operator based on the hypervolume measure.[13d] The design of hypervolume-based MOEAs has triggered an important amount of research, because such approaches scale better than Pareto ranking when increasing the number of objectives. However, computing the hypervolume is a computationally expensive task, and this has limited its use.

Regarding diversity maintenance, there have been several proposals in the specialized literature. The most popular approaches are fitness sharing and niching,[14] clustering,[15] crowding,[10] geographically based schemes,[16] and the use of entropy.[17] In all of them, the main idea is to favor the exploration of regions of search space in which there are less solutions. The density of solutions can be measured either in decision variable space or in objective function space (or even in both). Additionally, some researchers have proposed the use of mating restriction schemes as a way of preserving diversity.[8]

A third component of modern MOEAs is *elitism*, which normally consists of using an external archive (called a 'secondary population') that can (or cannot) interact in different ways with the main (or 'primary') population of the MOEA. The main purpose of this archive is to store all the nondominated solutions generated throughout the search process, while removing those that become dominated later in

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the search (called local nondominated solutions). The approximation of the Pareto optimal set produced by a MOEA is thus the final contents of this archive. The use of elitism is important, since this mechanism is required to guarantee convergence of a MOEA, from a theoretical perspective.[3]

The high number of publications on EMO currently available makes evident that this research area is still very active. Current publications report not only a wide variety of new applications (e.g., see Ref 18), but also important algorithmic developments, as well as research on more specialized topics (e.g., incorporation of user's preferences, surrogate methods, theoretical foundations, approaches for dealing with problems having many objectives, new ranking methods, new constraint-handling techniques, use of alternative metaheuristics, etc.).[3] Nevertheless, and in spite of the (somewhat intimidating) high number of existing publications, there is still plenty of room for newcomers (either students or researchers) as well as for more practitioners. In fact, the main aim of this paper is precisely to attract the interest of more people toward this research area, which is not only exciting but also widely applicable.

NOTES

[a] A *metaheuristic* is a high level strategy for exploring search spaces by using different methods.[2] Metaheuristics have two main procedures: one for diversification (i.e., exploration of the search space) and one for intensification (i.e., exploitation of the accumulated search experience).
[b] The author maintains the EMOO repository, which currently contains over 5800 bibliographic references related to evolutionary multiobjective optimization. The EMOO repository is available at: http://delta.cs.cinvestav.mx/~ccoello/EMOO/.
[c] Without loss of generality, we will assume only minimization problems.
[d] The *hypervolume* (also known as the *S* metric or the Lebesgue measure) of a set of solutions measures the size of the portion of objective space that is dominated by those solutions collectively.

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