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Optimal design of planar and space structures with genetic algorithms

Fuat Erbatur*, Oğuzhan Hasançebi, İlker Tütüncü, Hakan Kılıç

Department of Civil Engineering, Middle East Technical University, 06531 Ankara, Turkey

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Abstract

One of the most important practical considerations in the optimization of steel structures is that structural members are generally to be selected from available steel profiles. Furthermore, the solutions produced according to the national specifications are undoubtedly valuable from the point of view of applicability in real-life practice. This paper reports the development of a computer-based systematic approach for discrete optimal design of planar and space structures composed of one-dimensional elements. The main characteristic of the solution methodology is the use of a genetic algorithm (GA) as the optimizer. Applications and experience on steel frame and truss structures are discussed. The results of comparative studies of the GA against other various discrete and continuous optimization algorithms for a class of representative structural design problems are reported to show the efficiency of the former. It is observed that a GA often finds the region of the search space containing the global optimum, but not the true optimum itself. Also, in this study an approach based on a proposed multilevel optimization is tested and proved to overcome this shortcoming. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Optimization; Structural optimization; Steel structures; Integrated genetic algorithm; Constraint handling; Computer program 'GAOS'; Multilevel optimization

1. Introduction

This paper is concerned with a computer-based systematic approach for optimum design of planar and space structures composed of one-dimensional elements. In structural optimization it is well-known that optimality criteria (OC) techniques and mathematical programming (MP) based methods have received great interest and application during the last several decades. Recently, genetic algorithms (GA), simulated annealing and evolutionary programming have attracted attention amongst the engineering design optimization community [1]. These new approaches show certain advantages over the more classical optimization procedures [2], e.g. they can successfully be applied to a broad range of diverse problem areas. In this study design optimization approach using a GA is employed. Genetic algorithms are global search methods which have found application in a wide spectrum of problem areas, including optimum structural design [3–7]. Here, discrete optimal designs of bar structures including beams, plane and space trusses, plane and space frames and grids is covered using a GA based procedure. Discrete variable optimization refers to the case where the design variables are to be selected from an available list. Integer and 0–1 optimizations are easily inter-

* Corresponding author. Fax: +90-312-2101-262.
E-mail address: eko@metu.edu.tr (F. Erbatur).

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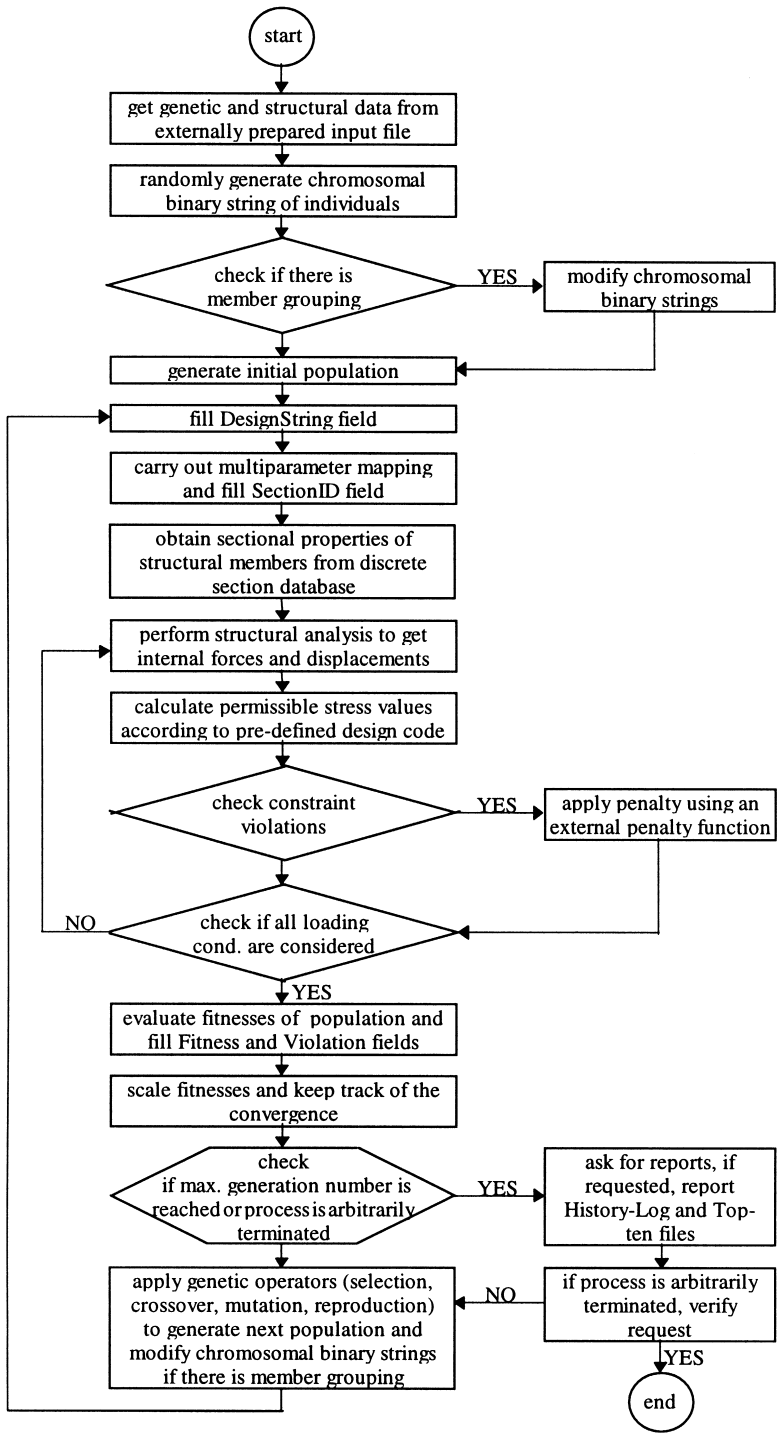

Fig. 1. General flowchart of GAOS.

preted as two special cases of the general discrete variable optimization approach. The need for more research on discrete variable optimization is pointed out in Thanedar and Vanderplaats [8], where the three principal methods for discrete variable optimization are stated as the classical branch and bound methods, mixed linearization or approximation methods and ad-hoc methods. Included in the ad-hoc methods are simulated annealing and genetic algorithms.

The integrated GA approach transforms the constrained structural design problem into an unconstrained problem through the use of penalty functions. Recent penalty functions for GA applications are discussed in Michalewicz [9]. In this paper, a slightly modified version of the Joines and Houck's method is used. The GA is a robust optimizer. A proper treatment of GA tools [11] allows both a thorough exploration of the design space and a satisfactory exploitation of the already obtained possible solutions in the search of the optima. A global optimum is not guaranteed, although, near-optimal solutions are found easily. In the weight optimization problems presented in this paper, discrete optimization is performed by selecting discrete sections of structural elements from available profile lists. Although only size optimization of bar structures is considered, GA integrated structural optimization can conceivably handle other structural types and more complex structures. For carrying out the necessary computations, a computer program GAOS (Genetic Algorithm Based Optimum Structural Design) is introduced. The general flowchart of GAOS is given in Fig. 1. The main components of the system are: (1) structural modeling (displacement finite element structural analysis algorithm); (2) modeling for optimum design problem (objective function, design variables and constraints involved in size optimization); and (3) the optimization algorithm (GA).

The paper first discusses the basic working scheme of a GA and the GA integrated structural optimization. Then the implementation of the GAOS program is discussed in detail with respect to discrete size optimization of trusses and frames with applications to steel structures composed of ready hot-rolled sections. Six example problems are given and fully discussed.

2. Structural optimization and GA

The main three components in the operation of a GA are: (1) the creation of an initial pool of designs; (2) combination of the designs in a pool in order to produce better designs; and (3) obtaining new generations of designs. These steps are explained below in the context of structural design optimization problems.

2.1. Creation of a pool of designs

In GA terminology a pool of designs is named a population. Each design in a population is called an individual and the size of a population is the number of individuals which constitute the population. The individuals are defined by binary strings (there may be other representations) which represent the design parameters of a particular design. To start the algorithm, an initial population is created randomly.

2.2. Combination of designs

If the size of the population is n, $n/2$ pairs of designs are combined to produce n new designs, each pair yielding (producing) a new pair. In this process, natural evolution of living organisms is mimicked. Prior to the combination of individuals, there is a process of selection. Each individual is first evaluated with respect to the objective function and then assigned a fitness value. Those individuals with high fitnesses are selected for reproduction. A very fit individual may have several chances to be a mate in the reproduction phase. Selection, crossover and mutation are the most common genetic operators which are used in obtaining improved results.

2.3. New generation of designs

The end product of combination of $n/2$ pairs of the fittest designs is a new pool of designs having n new individuals. In GA terminology this is a new generation. The new generation has better fit individuals as compared to the previous generation. The underlying philosophy in a GA is that every time by starting the combination process with more fit individuals, the average fitness of the population should increase. In terms of a structural optimization problem, an individual represents a possible design, comprising mainly the relevant information related to the structure. Since a population consists of n individuals, this implies that each generation results in n designs. As new generations are created, and as it is explained above, the average fitness which represents the objective function of the optimum design problem increases which in turn means the optimum solution is approached.

3. Structural optimization problem

A general discrete-sizing structural optimization problem is posed as,

minimize

$$W = F(\boldsymbol{X}) \tag{1}$$

satisfying

$$g_j(\boldsymbol{X}) \leq 0, \quad j = 1, \ldots, N_c, \tag{2}$$

$$(X_i)_{\min} \leq X_i \leq (X_i)_{\max}, \quad i = 1, \ldots, N_d \tag{3}$$

where

$$\boldsymbol{X} = \left[X_1, X_2, \ldots, X_{N_d}\right]^{\mathrm{T}} \quad \text{and} \quad X_i \in \boldsymbol{L} \tag{4}$$

In Eqs. (1)–(4), N_c and N_d represent, respectively, the number of constraints and independent design variables, $\boldsymbol{X}$ is the vector of design variables of dimension $(N_d \times 1)$, $W = F(\boldsymbol{X})$ is the objective function, g_j is the jth constraint on structural response and the inequalities in Eq. (3) are side constraints on the design variables. In discrete structural engineering optimization the vector $\boldsymbol{X}$ represents ready element sections which are to be chosen from an available list ($\boldsymbol{L}$). Eqs. (1)–(4) represent a typical constrained optimization problem. The structural response constraints considered in this study include section stresses, nodal displacements and buckling of elements.

3.1. Integration with GA

For GA use, the constrained problem is transformed into an unconstrained problem (or GA is to be modified). A new (or modified) function is defined where the constraints that are violated, are penalized. Some of such approaches are studied in Michalewicz [9]. Amongst them the one proposed by Joines and Houck (Eq. (5)) uses a dynamic penalty with respect to generation count t.

$$\phi(\boldsymbol{X}) = F(\boldsymbol{X}) + (r_0 \times t)^{\alpha} \sum_{j=1}^{m} g_j^{\beta}(\boldsymbol{X}) \tag{5}$$

In Eq. (5), g_j is the jth constraint function, which is zero in case of no violation, and is positive otherwise. The $(r_0 \times t)^{\alpha}$ component of the penalty term is the penalty multiplier, where r_0 stands for the penalty coefficient. For r_0,α and β, the values of 0.5, 2 and 2 are recommended, respectively. In this paper, a slightly modified version of the Joines and Houck method (Eq. (6)), is used.

$$\phi(\boldsymbol{X}) = F(\boldsymbol{X}) + (r_0 \times t)^{2} \sum_{j=1}^{m} \left[g_j(\boldsymbol{X})k\right] \tag{6}$$

As it is noticed, the $g_j^{\beta}(X)$ term in Eq. (5) is replaced by $g_j(\boldsymbol{X})k$ in Eq. (6). In structural optimization problems, the constraints are normalized to provide an equal and fair penalty distribution for different types of constraint violations. When the power of the constraints are imposed, as in Eq. (5), the penalty given to the individuals becomes very small due to the normalization of the constraints. The modification in Eq. (6) avoids this drawback observed in Eq. (5). In Eq. (6), a value of 10 is used for k.

The handling of constraints in GA integrated structural optimization is an important issue in itself. This problem is studied in detail elsewhere [10], It suffices to note here that, in Ref. [10], apart from other considerations, the experience gained by employing Eq. (6) is also utilized and a new penalty function, Eq. (7), is proposed to obtain an improved search over the design space. Two new parameters are introduced in Eq. (7); a penalty parameter 'p' and a control parameter 'c'. In brief, the penalty parameter is designed to adjust the intensity of penalty imposed to constraints as generations progress and, the control parameter puts a control on the distribution of individuals in the design space. As stated above, in this paper Eq. (6) is used. Some of the numerical examples are also tested using Eq. (7) with encouraging results, i.e., optimum designs are reached in lesser number of generations. A full treatment of the problem of constraint handling is reported in Ref. [10].

$$\phi(\boldsymbol{X}) = F(\boldsymbol{X}) + c \cdot p \sum_{j=1}^{m} \left[g_j(\boldsymbol{X})k\right] \tag{7}$$

4. Genetic algorithm based structural optimization (GAOS)

The integrated structural optimization explained above is implemented as the program GAOS. The GAOS program is conceived to handle the optimal design of structures made up of 1D elements. In its present form it handles size optimization of plane and space trusses, plane and space frames, beams and grids with given topology and geometry. The general characteristics of the program are:

1. Constraint violations are taken care of by penalty functions.
2. A structural analysis program is included in the main program to achieve a shorter computation time.
3. The program requires the preparation of an input file for both the structural analysis problem and the GA required data. The GA data is user dependent. If not prepared the program will assume default values which make the program independent of the GA optimization procedure. The structural data is typical of a generally available structural analysis program.
4. For steel structures the program selects ready sec-

Individual #	DesignString	SectionID	Fitness	Violation
Individual (1)	0110110011	[13,76,..,14]	2723	FALSE
Individual (2)	1101110101	[3,12,..,104]	1714	TRUE
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....
Ind. (pop.size)	0110011110	[103,7,. .,98]	3112	TRUE

Fig. 2. An illustration of a typical population in GAOS.

tions from particular profile lists specified in the input data.

Apart from these general characteristics, the GAOS program has the following practical approaches.

1. Presently, it handles the requirements of both AISC and Turkish codes.
2. Member grouping, as widely practical in real design problems, is possible. This also decreases the number of design variables and thus the problem size.
3. Five different profile lists may be considered.
4. It allows the user to impose his own constraints.
5. It allows the user to establish his own profile lists.
6. Multiple loading conditions can be handled.

The outline of the GAOS implementation is presented in the following sections.

4.1. Definition of an individual

The individuals of the populations are Pascal record types and contain DesignString, SectionID, Fitness and Violation fields, where the characteristics of individuals are stored, Fig. 2. DesignString field represents the chromosome, and holds the binary string which is a combination of individual strings corresponding to members in the structural system. Mapping of the chromosomal strings to discrete sections is carried by multiparameter mapping. Multiparameter mapping is found to be easy to handle and implement for structural design problems. SectionID field of the individuals holds the discrete section numbers corresponding to structural members. SectionID variable is an array of integers which is found by decoding and mapping the DesignString. Discrete sections are found by using the entries of SectionID array as indexes into the corresponding profile list holding the discrete sections. After the fitness of each individual in a population is evaluated, the value is kept in the Fitness field for use in the creation of the next generation. The Violation field is a Boolean variable used to indicate whether there is any constraint violation or not. The Violation field is set to TRUE when there is a violation, and FALSE otherwise.

4.2. Profile lists and database for discrete sections

GAOS can use all profile lists available in AISC and Turkish codes, such as circular hollow, wide-flange profiles, etc. Any profile list (prepared as a DOS file) is arranged to be a list of entries defining the characteristics of all discrete sections of a particular profile. A typical profile list with its five entries is shown in Table 1. In GAOS, the maximum number of discrete sections in a profile list is limited to 300, and a maximum five different profile lists are allowed to be used in each run of the program. When the program is implemented, the profile lists used in the problem are read and transferred to a database within GAOS to speed up the process. This database declared as a dynamic variable is of Pascal record type and contains Name, Height, Width, WebS, FlangeT, Area, UnitWeight, InertiaX,

Table 1
First five discrete sections in a typical wide-flange discrete section database

Name	Height, h (in.)	Width, w (in.)	WebS, s (in.)	FlangeT, t (in.)	Area (in.^{2})	Unit weight (lb/ft)	InertiaX (in.^{4})	IX (in.)	InertiaY (in.^{4})	IY (in.)	J (in.^{4})
W44 × 285	44.02	11.811	1.024	1.772	83.80	285.00	24600.0	17.10	490.0	2.42	60.0
W44 × 248	43.62	11.811	0.865	1.575	72.80	248.00	21400.0	17.20	435.0	2.44	40.7
W44 × 224	43.31	11.811	0.787	1.416	65.80	224.00	19200.0	17.10	391.0	2.44	30.0
W44 × 198	42.91	11.811	0.709	1.220	58.00	198.00	16700.0	16.90	336.0	2.41	20.1
W40 × 328	40.00	17.910	0.910	1.730	96.40	328.00	26800.0	16.70	1660.0	4.15	74.2

IX, InertiaY, IY, J fields, each holding a section property for an I-beam, Fig. 3.

In this record type, Name, Area and UnitWeight correspond, respectively, to the name, cross-sectional area and weight per linear length of the discrete section. Height is the height of the section, Width is the width of the flange. WebS and FlangeT refer to thickness of the web and flange, respectively. InertiaX and InertiaY are used to denote the moment of inertia of the section about the strong and weak axis, respectively, while IX and IY stand for the corresponding radii of gyration. Finally, J refers to the torsional constant of the discrete section.

4.3. Structural analysis and constraints

The structural analysis part has to be compatible with the genetic algorithm. The use of an external analysis program is proved to be inefficient; therefore an analysis program is incorporated into GAOS itself. The analysis routine handles both joint and span loads and restraints. Three typical constraints, namely stress, stability and displacement constraints can be imposed in GAOS. Presently, the user has the choice of selecting the AISC or the Turkish code specifications.

4.4. Fitness and penalty function

A penalty function is to be chosen for handling constraint violations during the fitness evaluation procedure. In case of violation of a constraint, a penalty associated with the constraint is applied to the current design. GAOS uses Eq. (6) to handle the constraint violations.

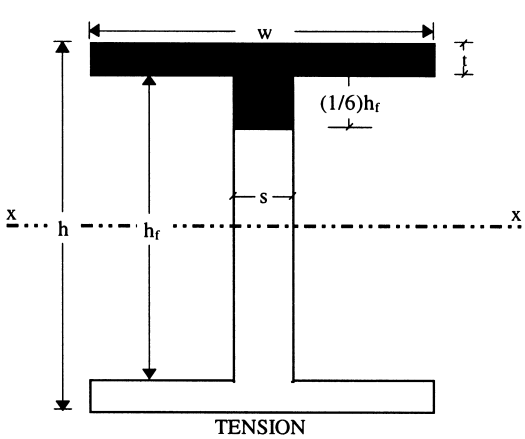

Fig. 3. I-beam.

4.5. Genetic operators

After the evaluation of current population's fitness, next population is created from the current one. Genetic operators are involved at this stage. Selection, reproduction and mutation operators, as implemented in Goldberg's Simple Genetic Algorithm source code [12], are used. Fitness Scaling is also used to help smooth convergence by narrowing the range of population fitnesses to gather around population average fitness. Individuals whose fitnesses are below the population average fitness are scaled up and individuals whose fitnesses are above the population average fitness are scaled down. In this way, dominance by extraordinary individuals is eliminated, and better convergence and better search of design space are established. The fitness scaling algorithm used in GAOS is shown below. In this algorithm, OriginalFit and ScaledFit are the variables showing the original and scaled fitness values of an individual, respectively. MinFit, AvgFit and MaxFit are the minimum, average and maximum population original fitness values, respectively. Additionally, the Elitist Selection Scheme (ESS) [2], is used so as not to lose the fittest individual during the creation of new populations.

Algorithm:

0.1. **Set** FitMult = 2.0 { Fitness multiple is 2 }
0.2. **Evaluate** MinFit, AvgFit and MaxFit of the population
0.3. **If** MinFit > (FitMult∗AvgFit−MaxFit)/(FitMult−1.0)
  then Δ = MaxFit−AvgFit {Normal Scaling }
    if Δ = 0.0 **then** a = 1.0, b = 0.0
      else a = (FitMult−1.0)∗AvgFit/Δ, b = AvgFit∗(MaxFit−FitMult∗AvgFit)/Δ

  else Δ = AvgFit−MinFit { Scale as much as possible }
    if Δ = 0.0 **then** a = 1.0, b = 0.0
      else a = AvgFit/Δ, b = −MinFit∗AvgFit /Δ

0.4. **Compute** ScaledFit = a∗OriginalFit + b

4.6. Termination criterion and top-ten

Considering the advantage of handling a population of designs in each step, a general termination criterion

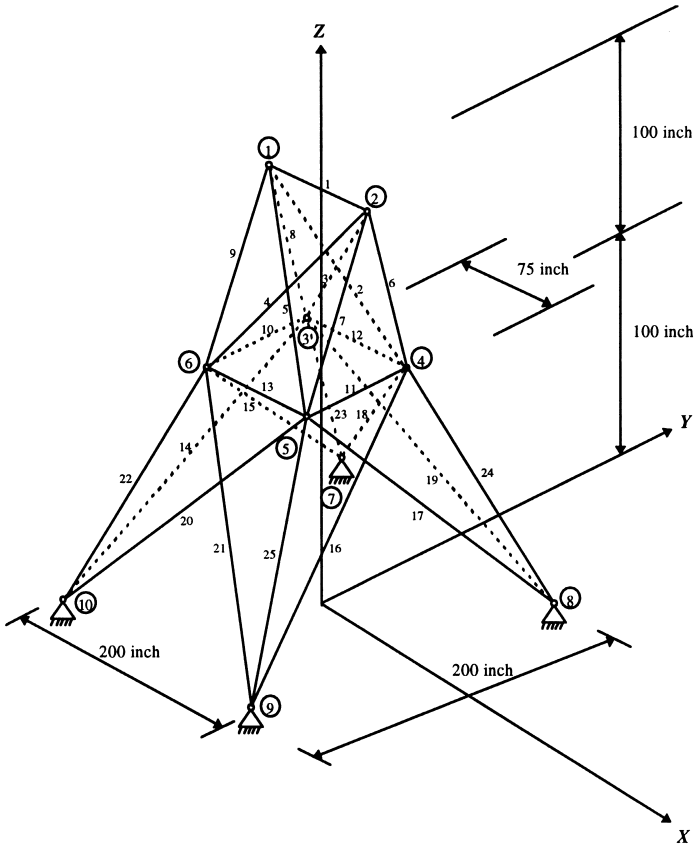

Fig. 4. A 25-bar space truss.

is not enforced in GAOS. Instead the termination is left to the user. On termination, two valuable pieces of information may be obtained; History-Record and a predetermined number of feasible designs. The History-Record file enables the user to view what happens during consecutive generations of the process through statistical treatment of the individuals. It contains information on the feasible (and infeasible) best, average and worst designs as the generations progress. Top-ten refers to the best ten different designs obtained during the optimization process until it is terminated. Each design includes the objective function value, the section chosen for each element and the analysis results for the applied loading.

5. Use of GAOS

For the operation of GAOS, one has to prepare an input file where in addition to typical structural analysis input options, data related to genetic operations are also included. These are: population size, maximum number of generations, mutation probability, and crossover probability. Additionally, two more data are also considered. One is a penalty coefficient, which is needed in tackling constraint violations and the other one is to indicate the element group to which individual elements belong. The use of the program is facilitated by an interactive operation where the main components of the optimization procedure appear on

Table 2
Member linking detail for 25-bar space trus

Group number	Members
1	A_1
2	A_2–A_5
3	A_6–A_9
4	A_{10}, A_{11}
5	A_{12}, A_{13}
6	A_{14}–A_{17}
7	A_{18}–A_{21}
8	A_{22}–A_{25}

Table 3
The design data for 25-bar space truss

Constraint data				
Displacement constraints: $\Delta_j \leq 0.35$ in. in x and y directions, $j = 1,2$ Stress constraints: -40 ksi $\leq (\sigma_a)_i \leq 40$ ksi $i = 1,\ldots,25$				
Loading data				
Case number	Joint number	x (kips)	y (kips)	z (kips)
1	1	1	−10	−10
	2	0	−10	−10
	3	0.5	0	0
	6	0.6	0	0
Elements of discrete sets (in.^{2})				
0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4				
Material properties				
Modulus of elasticity: 10^4 ksi Density of material: 0.1 lb/in.^{3}				

the computer screen. The user chooses the input file and starts the process. Appearing on the screen at any time are current design values (with weight, volume, and constraint status), sections of the members, and number of generations. Additionally, input file name, number of members and the chosen design code also continuously appear on the screen.

6. Multilevel optimization

6.1. A drawback of a GA

Optimization of a structural system comprising numerous design variables is a challenging problem due to its huge size of search space. In fact, it is not easy for any optimization algorithm to carry out an effective exploration without locating a local optimum. The dimension of the search space even grows exponentially either with the addition of extra design variables or with the enlargement of profile lists. The tests performed on various structures show that a GA is generally quite successful in locating the region of the search space containing the global optimum, but not the true optimum itself.

Table 5
Member linking detail for 72-bar transmission tower

Group number	Members
1	A_1–A_4
2	A_5–A_{12}
3	A_{13}–A_{16}
4	A_{17}, A_{18}
5	A_{19}, A_{22}
6	A_{23}–A_{30}
7	A_{31}–A_{34}
8	A_{35}, A_{36}
9	A_{37}–A_{40}
10	A_{41}–A_{48}
11	A_{49}–A_{52}
12	A_{53}, A_{54}
13	A_{55}–A_{58}
14	A_{59}–A_{66}
15	A_{67}–A_{70}
16	A_{71}, A_{72}

6.2. Multilevel optimization approach

Here, a multilevel optimization approach is implemented and proved to eliminate the effect of such a major drawback. The approach rests on reducing the size of the search space for individual design variables in each successive level of the optimization process. In this approach, an initial optimization named the first

Table 4
The comparison table for 25-bar space truss

Method	Weight (lb)	Design variables (in.^{2})							
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
Rajeev [4] GA, pop. size = 20	546.76	0.20	1.80	2.30	0.20	0.10	0.80	1.80	3.00
Rajeev [4] GA, pop. size = 30	546.01	0.10	1.80	2.30	0.20	0.10	0.80	1.80	3.00
Rajeev [4] GA, pop. size = 40	546.01	0.10	1.80	2.30	0.20	0.10	0.80	1.80	3.00
Zhu [13]	562.93	0.10	1.90	2.60	0.10	0.10	0.80	2.10	2.60
GAOS-level1	515.27	0.10	1.00	3.40	0.20	0.60	1.10	0.90	3.00
GAOS-level2	493.80	0.10	1.20	3.20	0.10	1.10	0.90	0.40	3.40

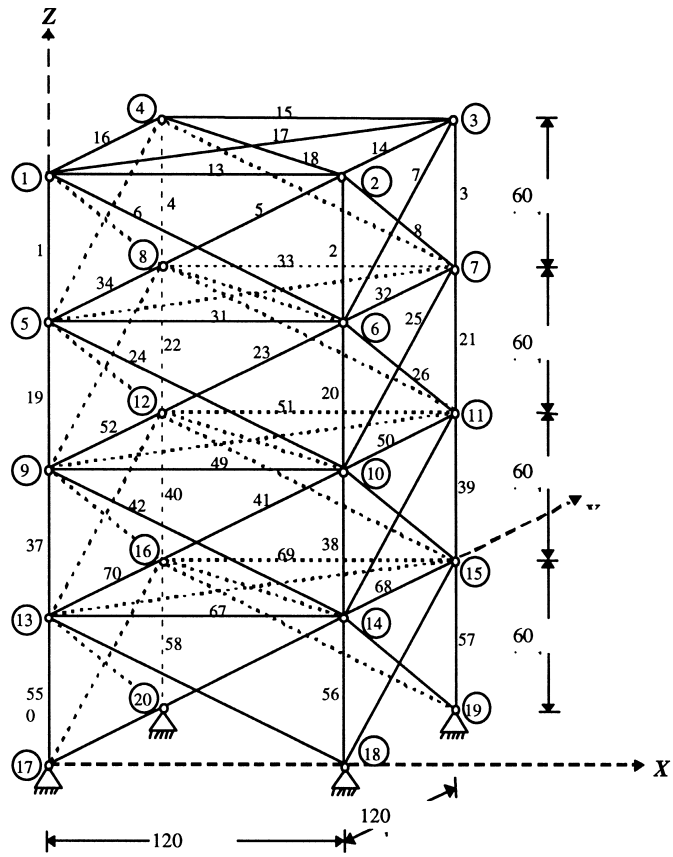

Fig. 5. A 72-bar transmission tower.

level optimization is carried out with an original profile list (discrete set) used by all design variables. An original discrete set must be arranged in such a manner that the ready sections are placed in an increasing order of their cross-sectional areas. In other words, the first entry of the discrete set is the smallest section, and the last entry is the largest one. Following the first level optimization, the algorithm automatically divides the original discrete set into several subsets (sub-profile lists with smaller size search spaces) to be employed in the second level optimization. The procedure used to create these subsets is as follows; (i) the best design obtained in the first level optimization is taken as the reference design, (ii) the original discrete set is equally divided into a prescribed number of subsets (generally 5), (iii) individual design variables are directed to appropriate subsets according to their values obtained in the reference design, and (iv) the enlargement of the subsets is performed. For this, each subset is added to some ready sections copied from neighbour subsets, that is the last 1/5 sections of the previous and the first 1/5 sections of the following neighbour subsets are added as the initial and final entries to this subset, re-

Table 6
The design data for 72-bar transmission tower

Constraint data				
Displacement constraints: $\Delta_j \leq 0.25$ in. in x and y direction $j = 1,\ldots,20$ Stress constraints: -25 ksi $\leq (\sigma_a)_i \leq 25$ ksi $i = 1,\ldots,72$ Member-size constraints: 0.1 in.^{2} $\leq A_i$, $i = 1,\ldots,72$				
Loading data				
Case number	Joint number	x (kips)	y (kips)	z (kips)
1	1	5	5	−5
2	1	0	0	−5
	2	0	0	−5
	3	0	0	−5
	4	0	0	−5
Material properties				
Modulus of elasticity: 10^4 ksi Density of material: 0.1 lb/in.^{3}				

Table 7
The comparison table for 72-bar transmission tower

Design variables (in.^{2})		Method						
Linked variables	Members	Venkayya [14]	Gellatly [15]	Renwei [16]	Schmit [17]	Xicheng [18]	GAOS level 2	GAOS level 3
A_1	1–4	0.161	0.1492	0.1641	0.1585	0.157	0.155	0.161
A_2	5–12	0.557	0.7733	0.5552	0.5936	0.537	0.535	0.544
A_3	13–16	0.377	0.4534	0.4187	0.3414	0.411	0.480	0.379
A_4	17,18	0.506	0.3417	0.5758	0.6076	0.571	0.520	0.521
A_5	19–22	0.611	0.5521	0.5327	0.2643	0.509	0.460	0.535
A_6	23–30	0.532	0.6084	0.5256	0.5480	0.522	0.530	0.535
A_7	31–34	0.100	0.1000	0.1000	0.1000	0.100	0.120	0.103
A_8	35,36	0.100	0.1000	0.1000	0.1509	0.100	0.165	0.111
A_9	37–40	1.246	1.0235	1.2893	1.1067	1.286	1.155	1.310
A_{10}	41–48	0.524	0.5421	0.5201	0.5793	0.516	0.585	0.498
A_{11}	49–52	0.100	0.1000	0.1000	0.1000	0.100	0.100	0.110
A_{12}	53,54	0.100	0.1000	0.1000	0.100	0.100	0.100	0.103
A_{13}	55–58	1.818	1.464	1.9173	2.0784	1.905	1.755	1.910
A_{14}	59–66	0.524	0.5207	0.5207	0.5034	0.518	0.505	0.525
A_{15}	67–70	0.100	0.1000	0.1000	0.1000	0.100	0.105	0.122
A_{16}	71,72	0.100	0.1000	0.1000	0.1000	0.100	0.155	0.103
Weight (lb)		381.2	395.97	379.66	388.63	380.84	385.76	383.12

spectively. This is necessary because some design variables might be placed in a subset whose upper or lower bound exactly matches with the value of the design variable. In such a case, the design variable is restricted to move in one direction, and thus may easily get trapped in a local optimum. However, such a treatment of subsets avoids the risk of locating a local optimum. In the second level optimization, the design variables use smaller size subsets. Hence, the second level optimization is performed on more restricted regions of the search space. The process continues in a similar fashion by dividing the subsets into new subsets and directing the design variables to the most appropriate search spaces.

Table 8
The design data for 112-bar steel dome

Constraint data				
Displacement constraints: $\Delta j \leq 20$ mm in z direction, $j = 1,17,23$ Stress constraints: $(\sigma t)_i \leq 165$ N/mm^2, $i = 1,\ldots,112$,$(\sigma_c)_i \leq$ TURKISH spec., $i = 1,\ldots,112$ Member-size: constraints: TURKISH circular hollow section $i = 1,\ldots,112$				
Loading data				
Case number	Joint number	x (kg)	y (kg)	z (kg)
1	1	0	0	−500
	17,23,29,35	0	0	−40
	16,18,22,24 28,30,31,32	0	0	−120
	Rest	0	0	−200
Material properties				
Modulus of elasticity 2.1E6 kg/cm^2 Yield strength 2750 kg/cm^2				

6.3. Characteristics of the multilevel optimization

The proposed approach has two important characteristics. Firstly, it encourages the optimization process to investigate better solutions in more restricted favourable regions of the search space. Therefore, each

Table 9
The comparison table for 112-bar steel dome

Volume (cm^3)	Design variables (mm^2)						
	A_1	A_2	A_3	A_4	A_5	A_6	A_7
Saka [19] 444700	714	778					
GAOS-level2 435724	707	557	667	707	707	523	1120

optimization level may be interpreted as one step of climbing up a hill towards the summit (optimum). Also, it performs well in case the search space is occupied by closely placed optima. Secondly, since the capacity of a discrete set is kept constant, any subset formed by dividing a discrete set must contain fewer ready sections than its capacity. That means excess slots are produced in the subsets. For continuous optimization problems, these slots can be utilized to obtain a better approximation to the continuous solution. During the tests, it is observed that two and three levels of optimization are adequate for the convergence to the true optimum for discrete and continuous optimization, respectively.

7. Numerical examples

Several standard test problems are used to verify the correctness and efficiency of the GAOS program in optimization of plane and space steel structures. In all problems considered, structures are designed for the minimum weight with the cross-sectional areas of

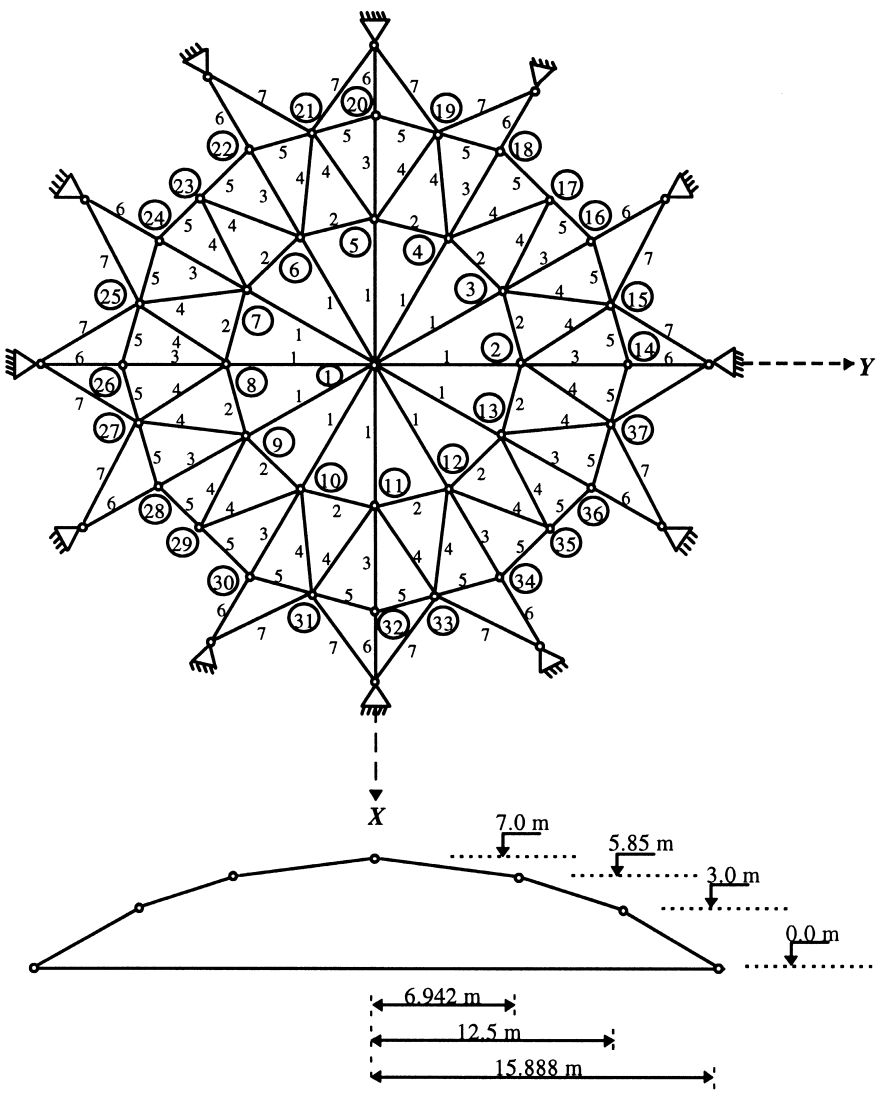

Fig. 6. A 112-bar steel dome.

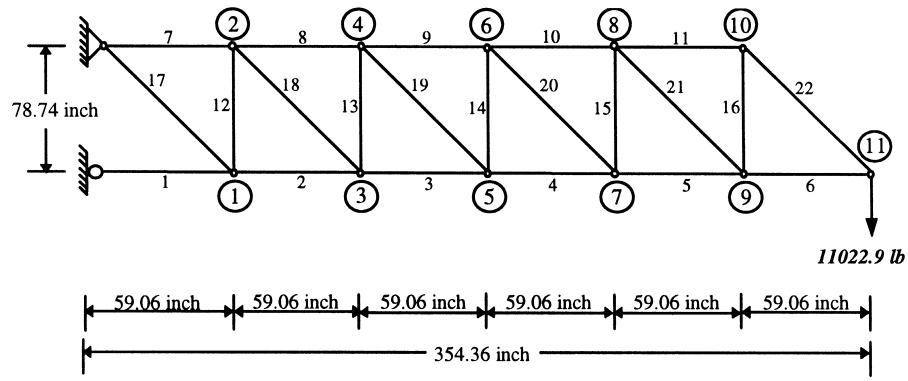

Fig. 7. Statically determinate 22-bar plane truss.

structural members being the design variables. The multilevel optimization approach discussed above is implemented in all the problems. Ten thousand structural analyses are performed in each optimization level of the problems tested here. For each problem, full list of design data, final designs at the end of optimization levels and comparison with other methods are presented.

Table 10
The comparison table for statically determinate 22-bar plane truss

Truss member	Axial force (kip)	Applied stress (ksi)	Allowable stress (ksi)	True optimum		GAOS level2		Optimum or not
	SAP90			Profile	Area (in.^{2})	Profile	Area (in.^{2})	
1	−49.607	−18.51	−18.85	P 31/2	2.68	P 31/2	2.68	OPT.
2	−41.339	−15.43	−18.85	P 31/2	2.68	P 31/2	2.68	OPT.
3	−33.071	−14.83	−18.26	P 3	2.23	P 3	2.23	OPT.
4	−24.803	−14.59	−17.20	P 21/2	1.70	P 21/2	1.70	OPT.
5	−16.535	−15.45	−15.89	P 2	1.07	P 2	1.07	OPT.
6	−8.267	−10.35	−13.62	P 11/2	0.799	P 11/2	0.799	OPT.
7	41.339	18.54	21.60	P 3	2.23	P 3	2.23	OPT.
8	33.071	19.45	21.60	P 21/2	1.70	P 21/2	1.70	OPT.
9	24.803	16.76	21.60	EP 2	1.48	P 21/2	1.70	NOT OPT.
10	16.535	20.69	21.60	P 11/2	0.799	P 11/2	0.799	OPT.
11	8.267	19.09	21.60	EP3/4	0.433	P 11/4	0.669	NOT OPT.
12	−11.022	−10.30	−12.97	P 2	1.07	P 2	1.07	OPT.
13	−11.022	−10.30	−12.97	P 2	1.07	P 2	1.07	OPT.
14	−11.022	−10.30	−12.97	P 2	1.07	P 2	1.07	OPT.
15	−11.022	−10.30	−12.97	P 2	1.07	P 2	1.07	OPT.
16	−11.022	−10.30	−512.97	P 2	1.07	P 2	1.07	OPT.
17	13.779	21.56	21.60	EP 1	0.639	EP 1	0.639	OPT.
18	13.779	21.56	21.60	EP 1	0.639	P 11/2	0.799	NOT OPT.
19	13.779	21.56	21.60	EP 1	0.639	EP 1	0.639	OPT.
20	13.779	21.56	21.60	EP 1	0.639	EP 11/4	0.881	NOT OPT.
21	13.779	21.56	21.60	EP 1	0.639	EP 1	0.639	OPT.
22	13.779	21.56	21.60	EP 1	0.639	P 11/2	0.799	NOT OPT.
Weight (lb)					524.5		548.06	17 optimum
Volume (in.^{3})					1849.9		1932.21	

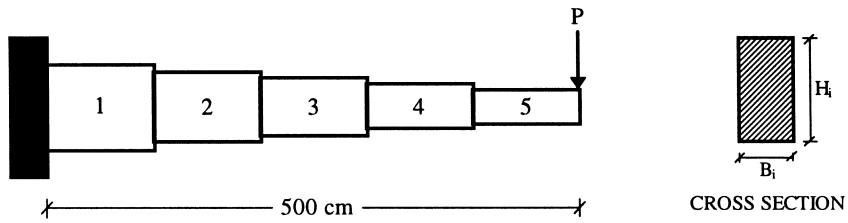

Fig. 8. Stepped cantilever beam.

Table 11
The design data for stepped cantilever beam

Constraint data	
Displacement constraint: tip deflection ≤ 2.7 cm in vertical direction Stress constraints: $(\sigma_b)_i \leq 14{,}000$ N/cm^2 $i = 1,\ldots,5$ Aspect ratio constraints, $H_i/B_i \leq 20$, $i = 1,\ldots,5$	
Loading data	
P	50,000 N
Material properties	
Modulus of elasticity	200 GPa
Sets of design variables (cm)	
B_1, H_1	Integer
B_2, B_3	2.4, 2.6, 2.8, 3.1
H_2, H_3	45.0, 50.0, 55.0, 60.0
B_4, H_4, B_5, H_5	Continuous

7.1. Example 1: 25-bar space truss

The standard 25-bar truss shown in Fig. 4 has been frequently used to test numerous optimization techniques [4,13]. Truss members linked in eight groups, as shown in Table 2, are to be selected from a discrete set of 30 ready sections. Table 3 presents complete and detailed list of design data. A comparison of optimal designs reported in the literature is listed in Table 4.

7.2. Example 2: 72-bar transmission tower

The design of 72-member transmission tower shown in Fig. 5 has an important place in continuous optimization literature [14–18]. The truss is subjected to two distinct loading conditions and has sixteen independent design variables after linking, as indicated in Table 5. Table 6 summarizes full design data as well as loading. A comparison of optimal designs reported in the literature is listed in Table 7.

7.3. Example 3: 112-bar steel dome

Fig. 6 shows the 112-bar steel dome that has been

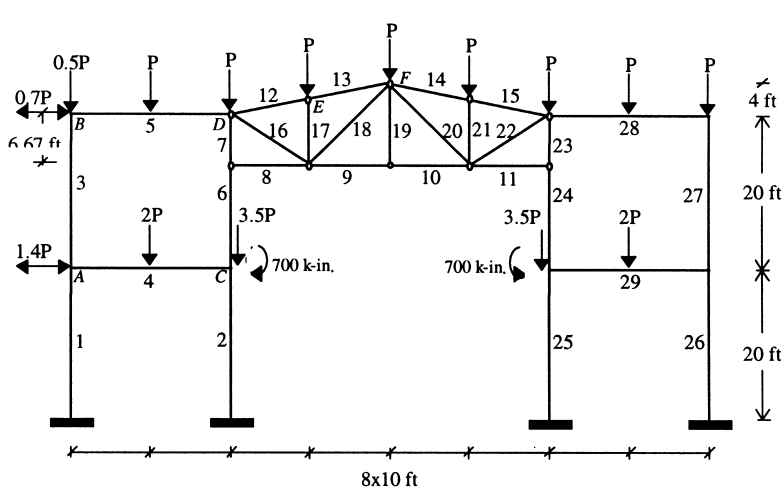

Fig. 9. Industrial building.

Table 12
The comparision table for stepped cantilever beam

Design variables (cm)	Method					
	Continuous/ round up	Precise discrete	Linear approximate discrete	Conservative approximate discrete	GAOS level 1	GOAS level 2
B_1	4	3	3	3	3	3
B_2	3.1	3.1	3.1	3.1	3.1	3.1
B_3	2.6	2.6	2.6	2.6	2.6	2.6
B_4	2.205	2.276	2.262	2.279	2.300	2.270
B_5	1.751	1.750	1.750	1.750	1.800	1.750
H_1	62	60	60	60	60	60
H_2	60	55	55	55	55	55
H_3	55	50	50	50	50	50
H_4	44.09	45.528	45.233	45.553	45.50	45.250
H_5	35.03	34.995	34.995	35.004	35.00	35.00
Volume (cm^3)	73555	64537	64403	64558	64815	64447

previously discussed in Saka [19]. Two modifications are made to the former treatment of the problem. First, 112 members of the dome are collected in seven distinct groups, whereas two groups are used in Saka [19]. Next, in place of AISC specification, the allowable compressive stress for each member is computed according to the Turkish specification. Despite the fact that the Turkish specification leads to safer values in computing the allowable compressive stresses, the distinction is not of much consequence. The complete and detailed list of design data is presented in Table 8, and a comparative study is listed in Table 9.

7.4. Example 4: statically determinate 22-bar plane truss

The statically determinant truss shown in Fig. 7 is considered to investigate how effectively and robustly the GA performs on structural optimization. Only AISC stress constraints are imposed on all truss members, for which circular hollow sections are adapted. Making use of statical determinacy of the truss, the true optimum solution is found precisely. No member linking is used to render the problem harder to optimize. Furthermore, the member forces are independent of member sizes due to statical determinacy of the truss. That means no compensation with the distribution of internal forces amongst the members is possible in case any design variable is exceeded. Table 10 shows and compares the true optimum value with that of GAOS, along with the structural response. When a third optimization level is employed, it is observed that the program finds the true solution precisely.

Table 13
Member linking detail for industrial building

Group number	Members
1	A_1, A_{26}
2	A_2, A_{25}
3	A_3, A_{27}
4	A_4, A_{29}
5	A_5, A_{28}
6	A_6, A_7, A_{23}, A_{24}
7	A_8, A_9, A_{10}, A_{11}
8	A_{12}, A_{13}, A_{14}, A_{15}
9	A_{16}–A_{22}
10	A_{17}–A_{21}
11	A_{18}–A_{20}
12	A_{19}

7.5. Example 5: stepped cantilever beam

Thanedar and Vanderplaats [8] applied various optimization techniques to optimize the stepped cantilever beam shown in Fig. 8. The beam is designed for minimum volume. The height and width in all five steps of the cantilever beam are chosen to be the design variables, i.e., 10 variables in all. Except for bending stress constraints, a specified aspect ratio is imposed such that the ratio of height to width in the steps of the beam is limited to be less than 20. Some design variables are set to be integer or continuous, while the others are to be selected from discrete sets. Thus, the problem shows a characteristic of integer, continuous and discrete behaviour. The complete and detailed design data is summarized in Table 11. Table 12 presents optimum values obtained by various methods.

Table 14
The design data for industrial building

Constraint data		
Displacement constraints, $\Delta_j \leq 1.5$ inch in horizontal direction at joints A, B, C, D, E, F for load cases 2 and 3 Stress constraints, -24 ksi $\leq (\sigma_t)_i \leq 24$ ksi $i = 1, \ldots, 29$, -16 ksi $\leq (\sigma_c)_i \leq 16$ ksi $i = 1, \ldots, 29$		
Loading data		
Case number	Load combinations (P = 10 kips)	
1	roof + floor + crane loads	
2	case I + wind load from left	
3	case I + wind load from right	
Sets of structural members		
Group number	Behaviour	Discrete set (AISC)
1, 2, 3, 4, 5, 6	Flextural members	Wide-flange (W)
7, 8	Flextural members	Tee section (WT)
9, 10, 11, 12	Axial members	Double-angle (L)
Material properties		
Modulus of elasticity	30×10^3 ksi	
Density of material	0.283 lb/in^3	
Yield stress	36 ksi	

7.6. Example 6: industrial building

Fig. 9 shows the geometry of an industrial building with the element numbering as well as the loading. It is subjected to three different loading conditions and has twelve design variables after linking, Table 13. The structure is composed of two side frames and a gable roof in between. The beam and column members of the side frames are to be designed as flexural members, for which wide flange sections are adapted. Similarly,

Table 15
The comparison table for industrial building

Design variables (in.^{2})		Method				
Linked variables	Members	Grierson and Lee [20] (in.^{2})	GAOS level 1		GAOS level 2	
			Profile	Area (in.^{2})	Profile	Area (in.^{2})
A_1	1, 26	9.13	W18 × 40	11.80	W16 × 36	10.60
A_2	2, 25	18.29	W18 × 55	16.20	W21 × 57	16.70
A_3	3, 27	4.42	W8 × 13	3.84	W6 × 16	4.74
A_4	4, 29	12.03	W14 × 43	12.60	W21 × 44	13.00
A_5	5, 28	6.46	W8 × 10	2.96	W6 × 12	3.55
A_6	6,7,23,24	9.13	W18 × 46	13.5	W14 × 26	7.69
A_7	8,9,10,11	2.20	WT3 × 4.5	1.34	WT3 × 4.5	1.34
A_8	12,13,14,15	2.20	WT5 × 8.5	2.50	WT3 × 4.5	1.34
A_9	16, 22	1.71	L3 × 2 × 1/4	2.38	L3 × 2 × 3/16	1.80
A_{10}	17, 21	1.09	L4 × 3.5 × 1/4	3.63	L3 × 2.5 × 3/16	1.99
A_{11}	18, 20	0.67	L2.5 × 2 × 3/16	1.62	L2.5 × 2 × 3/16	1.62
A_{12}	19	0.67	L3.5 × 2.5 × 5/16	0.31	L3 × 2 × 3/8	3.47
Weight (lb)		8991.6	9604.29		8693.21	

the top and bottom chord members of the gable roof show a flexural behaviour, whereas vertical and inclined web members of the gable roof only carry axial loads. Besides, the chord and web members are to be selected from tee sections cut from wide flange and two unequal-leg angles back to back sections, respectively. The full list of design data along with the loading conditions is presented in Table 14, and a comparative study between this work and Grierson and Lee [20] is shown in Table 15.

8. Discussion of results and conclusion

In this paper, the program GAOS was implemented for a class of typical structural design problems to confirm the ability of the GA on structural optimization. These problems, except for the statically determinate truss, have been used as test problems in the literature. Throughout the problems, the GA is set to compete with various continuous and discrete optimization algorithms. While the test problems 1, 3, 5 and 6 are in the category of discrete variable optimization, the test problems 2 and 5 show a fully and partially continuous behaviour, respectively. Comparing the tables, it can immediately be observed that the GA exhibits a satisfactory performance regardless of the problem type. More clearly, it found best solutions to test problem 1, 3 and 6. In test problems 2 and 5 it produced acceptably good values. Test problem 4 proved the power of the GA since 17 optimums among 22 variables were easily obtained in the second level optimization. The third level optimization found the true solution. An attractive attribute of the GA is that the optimization process does not need any gradient or other supplementary problem information. This, in fact, makes the optimization process easy with respect to more conventional techniques and explains why genetic algorithms are applied in a wide range of problem areas. The following two conclusions can be made from the above tests: (i) the GA is one of the most robust and promising strategies among discrete optimization techniques; and (ii) the multilevel optimization approach makes it possible for the GA to compete with continuous optimization techniques.

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